JEE Main 2021 | Kinematics and Vector Algebra

Question 1

(Only one correct answer) 2021

A stone is dropped from the top of a building. When it crosses a point $5\ m$ below the top, another stone starts to fall from a point $25\ m$ below the top. Both stones reach the bottom of building simultaneously. The height of the building is:

- (a) $35\ m$
- (b) $25\ m$
- (c) $50\ m$
- (d) $45\ m$

Question 2

(Only one correct answer) 2021

A bomb is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a :

- (a) hyperbola
- (b) straight line vertically down the plane
- (c) parabola in a direction opposite to the motion of plane
- (d) parabola in the direction of motion of plane

Question 3

(Only one correct answer) 2021

The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$. where α and β are constants and x & y are respectively the horizontal and vertical distance of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by:

- (a) $\tan^{-1}\alpha,\ \dfrac{4\alpha^2}{\beta}$
- (b) $\tan^{-1}\beta,\ \dfrac{\alpha^2}{2\beta}$
- (c) $\tan^{-1}\left(\dfrac{\beta}{\alpha}\right),\ \dfrac{\alpha^2}{\beta}$

(d) $\tan^{-1}\alpha, \dfrac{\alpha^2}{4\beta}$

Question 4

(Only one correct answer) 2021

Water drops are falling from a nozzle of a shower onto the floor from a height of $9.8\ m$. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

(a) $2.94\ m$

(b) $4.18\ m$

(c) $2.45\ m$

(d) $7.35\ m$

Question 5

(Only one correct answer) 2021

A helicopter is flying horizontally with a speed v at an altitude h has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped ?

(a) $\sqrt{\dfrac{2ghv^2+1}{h^2}}$

(b) $\sqrt{2v^2hg+h^2}$

(c) $\sqrt{\dfrac{2gh}{v^2}}+h^2$

(d) $\sqrt{\dfrac{2v^2h}{g}+h^2}$

Question 6

(Only one correct answer) 2021

Statement-I : Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $(\vec{P}\perp\vec{Q})$, when act at an angle θ_1 each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, then magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement-II : In the situation given above. $\theta_1 = 60^\circ$ and $\theta_2 = 90^\circ$

In the light of the above statement, choose the most appropriate answer from the options given below :

(a) Statement-I is true but Statement-II is false.

(b) Both Statement-I and Statement-II are true.

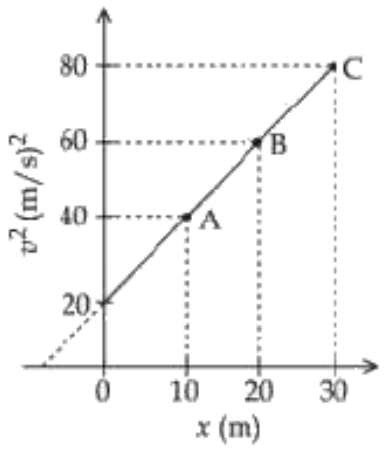
(c) Statement-I is false but Statement-II is true.

(d) Both Statement-I and Statement-II are false.

Question 7

(Integer type question) 2021

A particle is moving with constant acceleration $'a'$. Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is ..... m/s^2.



Question 8

(Only one correct answer) 2021

The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between $1\ s$ and $2\ s$ is:

(a) $\frac{\alpha}{2} + \frac{\beta}{3}$

(b) $\frac{3}{2}\alpha + \frac{7}{3}\beta$

(c) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

(d) $3\alpha + 7\beta$

Question 9

(Only one correct answer) 2021

A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is

(a) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$

(b) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

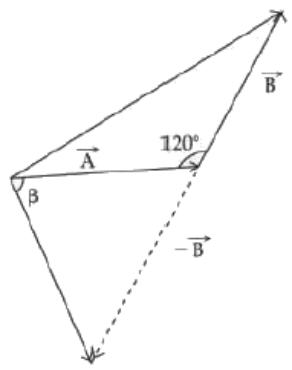
(c) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$

(d) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$

Question 10

(Only one correct answer) 2021

The angle between vector ($\overrightarrow{A}$) and ($\overrightarrow{A} - \overrightarrow{B}$) is :



(a) $\tan^{-1}\left(\frac{A}{0.7B}\right)$

(b) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

(c) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

(d) $\tan^{-1}\left(\frac{B\cos\theta}{A-B\sin\theta}\right)$

Question 11

(Only one correct answer) 2021

A force $\overrightarrow{F} = (40\hat{i} + 10\hat{j})\ N$ acts on a body of mass $5\ kg$. If the body starts from rest, its position vector $\overrightarrow{r}$ at time $t = 10\ s$, will be:

(a) $(100\hat{i} + 100\hat{j})\ m$

(b) $(400\hat{i} + 400\hat{j})\ m$

(c) $(100\hat{i} + 400\hat{j})\ m$

(d) $(400\hat{i} + 100\hat{j})\ m$

Question 12

(Only one correct answer) 2021

A ball is thrown up with a certain velocity so that it reaches a height h. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

(a) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(b) $\frac{1}{3}$

(c) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Question 13

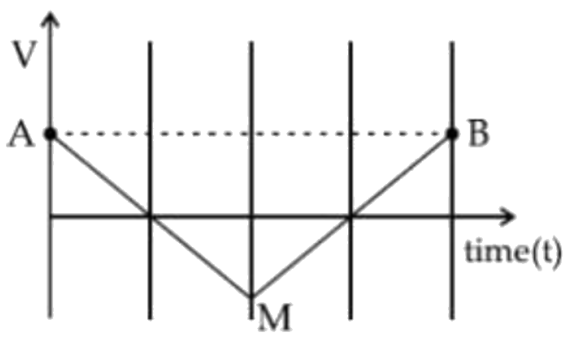
(Integer type question) 2021

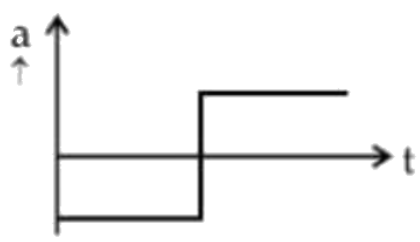
A bullet of mass $0.1\ kg$ is fired on a wooden block to pierce through it, but it stops after moving a distance of $50\ cm$ into it. If the velocity of bullet before hitting the wood is $10\ m/s$ and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is $'x'\ N$. The value of $'x'$ to the nearest integer is ..... .

Question 14

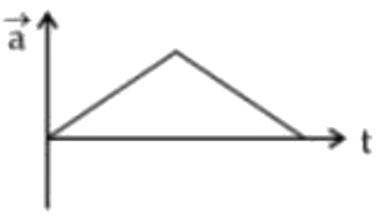
(Only one correct answer) 2021

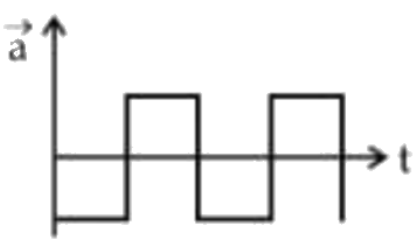
If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration - time graph ?



(a) 

(a) 

(a) 

(a) 

Question 15

(Only one correct answer) 2021

A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation $F = F_0\left[1 - \left(\frac{t-T}{T}\right)^2\right]$

Where F_0 and T are constants. The force acts only for the time interval $2T$. The velocity v of the particle after time $2T$ is :

- (a) $\frac{2F_0T}{M}$
- (b) $\frac{F_0T}{2M}$
- (c) $\frac{4F_0T}{3M}$
- (d) $\frac{F_0T}{3M}$

Question 16

(Only one correct answer) 2021

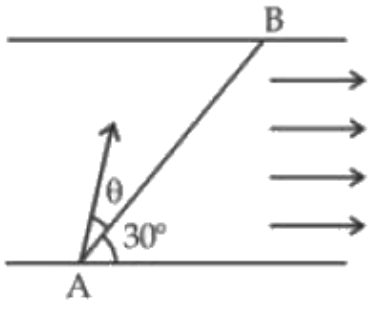
Consider two satellites S_1 and S_2 with periods of revolution $1\ hr.$ and $8\ hr.$ respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is :

- (a) $8:1$
- (b) $2:1$
- (c) $1:8$
- (d) $1:4$

Question 17

(Integer type question) 2021

A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ..... $^\circ$, so that the swimmer reaches point B.



Question 18

(Only one correct answer)

2021

An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point the train passes the signal post is

- (a) $\frac{v-u}{2}$
- (b) $\sqrt{\frac{v^2+u^2}{2}}$
- (c) $\sqrt{\frac{v^2-u^2}{2}}$
- (d) $\frac{u+v}{2}$

Question 19

(Integer type question)

2021

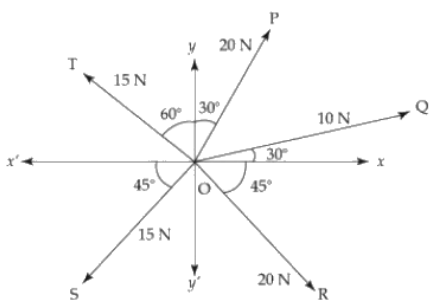
Two spherical balls having equal masses with radius of $5\ cm$ each are thrown upwards along the same vertical direction at an interval of $3\ s$ with the same initial velocity of $35\ m/s$, then these balls collide at a height of ..... m. (take $g = 10\ m/s^2$)

Question 20

(Only one correct answer)

2021

The resultant of these forces $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$, $\overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ..... N. [Take $\sqrt{3} = 1.7,\ \sqrt{2} = 1.4$. Given $\hat{i}$ and $\hat{j}$ unit vectors along $x,\ y$-axis]



- (a) $9.25\hat{i} + 5\hat{j}$
- (b) $3\hat{i} + 15\hat{j}$
- (c) $-1.5\hat{i} - 15.5\hat{j}$
- (d) $2.5\hat{i} - 14.5\hat{j}$

Question 21

(Only one correct answer)

2021

A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

(a) $t_2 - t_1$

(b) $\dfrac{t_1 t_2}{t_2 - t_1}$

(c) $\dfrac{t_1 t_2}{t_2 + t_1}$

(d) $\dfrac{t_1 + t_2}{2}$

Question 22

(Only one correct answer) 2021

A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\dfrac{t_1}{t_2}$ will be:

(a) $\dfrac{a_2}{a_1}$

(b) $\dfrac{a_1 + a_2}{a_1}$

(c) $\dfrac{a_1}{a_2}$

(d) $\dfrac{a_1 + a_2}{a_2}$

Question 23

(Integer type question) 2021

If the velocity of a body related to displacement x is given by $v = \sqrt{5000 + 24x}\ m/s$, then the acceleration of the body is ..... m/s^2

Question 24

(Only one correct answer) 2021

A player kicks a football with an initial speed of $25\ ms^{-1}$ at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take $g = 10\ ms^{-2}$)

(a) $h_{\max} = 15.625\ m,\ T = 1.77\ s$

(b) $h_{\max} = 3.54\ m,\ T = 0.125\ s$

(c) $h_{\max} = 10\ m,\ T = 2.5\ s$

(d) $h_{\max} = 15.625\ m,\ T = 3.54\ s$

Question 25

(Only one correct answer) 2021

Two vectors $\overrightarrow{X}$ and $\overrightarrow{Y}$ have equal magnitude. The magnitude of $(\overrightarrow{X} - \overrightarrow{Y})$ is n times the magnitude of $(\overrightarrow{X} + \overrightarrow{Y})$. The angle between $\overrightarrow{X}$ and $\overrightarrow{Y}$ is :

- (a) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$
- (b) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$
- (c) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$
- (d) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

Question 26

(Integer type question) 2021

A body of mass $2\ kg$ moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k})\ N$. It starts from rest and was at the origin initially. After $4\ s$, its new coordinates are $(8, b, 20)$. The value of b is ..... (Round off to the Nearest Integer)

Question 27

(Only one correct answer) 2021

The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is: (Where v stands for velocity)

- (a) $2mnv^3$
- (b) $2nv^3$
- (c) $2mv^3$
- (d) $2n^2v^3$

Question 28

(Only one correct answer) 2021

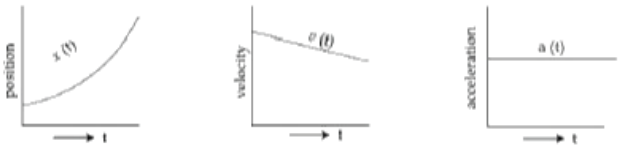
A balloon was moving upwards with a uniform velocity of $10\ m/s$. An object of finite mass is dropped from the balloon when it was at a height of $75\ m$ from the ground level. The height of the balloon from the ground when object strikes the ground was around. (Takes the value of g as $10\ m/s^2$)

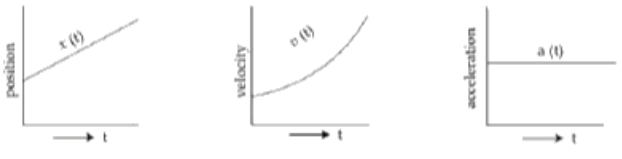
- (a) $125\ m$
- (b) $200\ m$
- (c) $300\ m$
- (d) $250\ m$

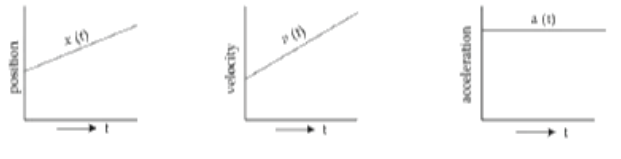
Question 29

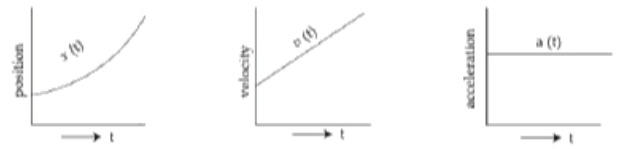
(Only one correct answer) 2021

The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :

(a) 

(a) 

(a) 

(a) 

Question 30

(Only one correct answer) 2021

A butterfly is flying with a velocity $4\sqrt{2}\ m/s$ in North-East direction. Wind is slowly blowing at $1\ m/s$ from North to South. The resultant displacement of the butterfly in 3 seconds is :

(a) $12\sqrt{2}\ m$

(b) $15\ m$

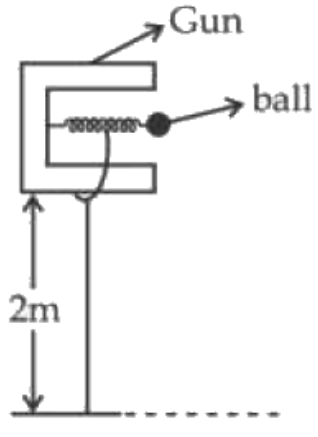
(c) $3\ m$

(d) $20\ m$

Question 31

(Integer type question) 2021

In a spring gun having spring constant $100\ N/m$ a small ball $'B'$ of mass $100\ g$ is put in its barrel (as shown in figure) by compressing the spring through $0.05\ m$. There should be a box placed at a distance $'d'$ on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of $2\ m$ above the ground. The value of d is ..... m. ($g = 10\ m/s^2$)



Question 32

(Only one correct answer) 2021

Two vectors $\overrightarrow{P}$ and $\overrightarrow{Q}$ have equal magnitudes. If the magnitude of $\overrightarrow{P}+\overrightarrow{Q}$ is n times the magnitude of $\overrightarrow{P}-\overrightarrow{Q}$, then angle between $\overrightarrow{P}$ and $\overrightarrow{Q}$ is:

- (a) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$
- (b) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$
- (c) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$
- (d) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

Question 33

(Only one correct answer) 2021

Water droplets are coming from an open tap at a particular rate. The spacing between droplets observed at 4^{th} second after its fall to the next droplet is $34.3\ m$. At what rate the droplets are coming from the tap? (Take $g=9.8\ m/s^2$)

- (a) 1 drops/seconds
- (b) 1 drops/7 seconds
- (c) 2 drops/seconds
- (d) 3 drops/2 seconds

Question 34

(Only one correct answer) 2021

The velocity of a particle is $v=v_0+gt+Ft^2$. Its position is $x=0$ then its displacement after time $(t=1)$ is :

- (a) $v_0+\frac{g}{2}+F$
- (b) $v_0+2g+3F$
- (c) v_0+g+F
- (d) $v_0+\frac{g}{2}+\frac{F}{3}$

Question 35

(Only one correct answer) 2021

A rubber ball is released from a height of $5\ m$ above the floor. It bounces back repeatedly, always

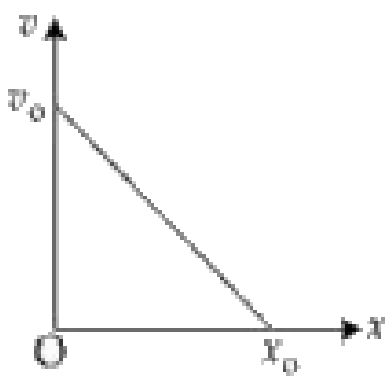
rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (take $g = 10\ ms^{-2}$)

- (a) $3.50\ ms^{-1}$
- (b) $2.0\ ms^{-1}$
- (c) $2.50\ ms^{-1}$
- (d) $3.0\ ms^{-1}$

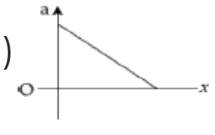
Question 36

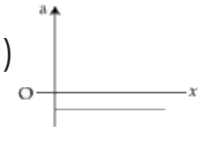
(Only one correct answer) 2021

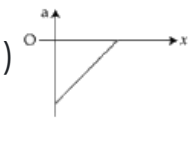
The velocity - displacement graph of a particle is shown in the figure.

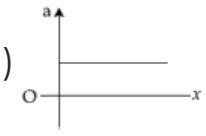


The acceleration - displacement graph of the same particle is represented by :

- (a) 

- (a) 

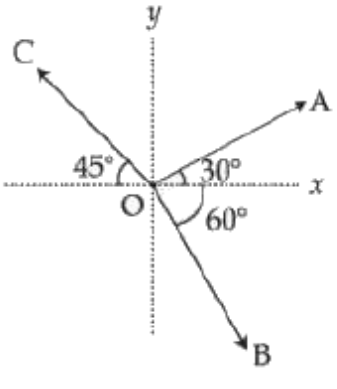
- (a) 

- (a) 

Question 37

(Only one correct answer) 2021

The magnitude of vector $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with x-axis will be:



- (a) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$
- (b) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$
- (c) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$
- (d) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

Question 38

(Integer type question) 2021

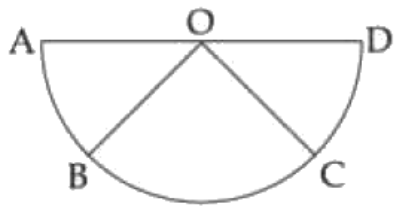
If $\overrightarrow{P} \times \overrightarrow{Q} = \overrightarrow{Q} \times \overrightarrow{P}$, the angle $\overrightarrow{P}$ and $\overrightarrow{Q}$ is $\theta\ (0^\circ < \theta < 360^\circ)$. The value of $'\theta'$ will be ..... $^\circ$.

Question 39

(Only one correct answer) 2021

Assertion A : If $A,\ B,\ C,\ D$ are four points on a semi-circular arc with center at $'O'$ such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$



In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) A is not correct but R is correct
- (b) Both A and R are correct but R is not the correct explanation of A
- (c) A is correct but R is not correct
- (d) Both A and R are correct and R is the correct explanation of A.

Question 40

(Integer type question) 2021

A person is swimming with a speed of $10\ m/s$ at an angle of 120° with the flow and reaches to a point directly opposite on the other side the river. The speed of the flow is $'x'\ m/s$. The value of $'x'$ to the nearest integer is .....

Question 41

(Only one correct answer) 2021

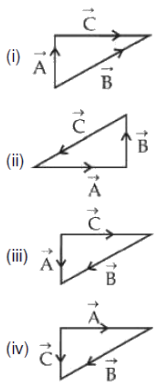
Match List-I with List-II

(a) $\vec{C}-\vec{A}-\vec{B}=0$

(b) $\vec{A}-\vec{C}-\vec{B}=0$

(c) $\vec{B}-\vec{A}-\vec{C}=0$

(d) $\vec{A}+\vec{B}=-\vec{C}$



Choose the correct answer from the options given below

- (a) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
- (b) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)
- (c) (a) → (i), (b) → (iv), (c) → (ii), (d) → (iii)
- (d) (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)

Question 42

(Integer type question) 2021

A swimmer can win with velocity of $12\ km/h$ in still water. Water flowing in river has velocity $6\ km/h$. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his staring point is .....°. (Round off to the Nearest Integer)

(Find the angle in degrees)

Question 43

(Only one correct answer) 2021

If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}\,.\,\vec{B}=|\vec{A}\times\vec{B}|$. Then the value of $|\vec{A}-\vec{B}|$ will be :

- (a) $\sqrt{A^2+B^2+2AB}$
- (b) $\sqrt{A^2+B^2-\sqrt{2}AB}$
- (c) $\sqrt{A^2+B^2}$
- (d) $\sqrt{A^2+B^2+\sqrt{2}AB}$

Question 44

(Only one correct answer) 2021

A mosquito is moving with a velocity $\vec{V}=0.5t^2\hat{i}+3t\hat{j}+9\hat{k}\ m/s$ and accelerating in uniform conditions. What will be the direction of mosquito after $2\ s$?

- (a) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis
- (b) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

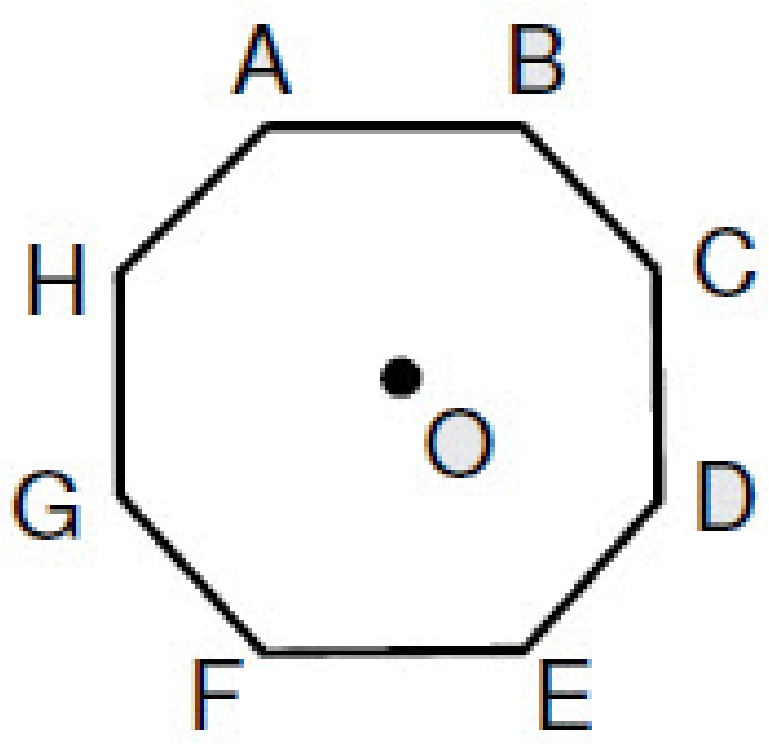
(c) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

(d) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

Question 45

(Only one correct answer) 2021

In an octagon $ABCDEFGH$ of equal side, what is the sum of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$. If $\overrightarrow{AO}=2\hat{i}+3\hat{j}-4\hat{k}$.



(a) $16\hat{i}+24\hat{j}+32\hat{k}$

(b) $16\hat{i}+24\hat{j}-32\hat{k}$

(c) $16\hat{i}-24\hat{j}+32\hat{k}$

(d) $-16\hat{i}-24\hat{j}+32\hat{k}$

Answer 1

Correct answers is D

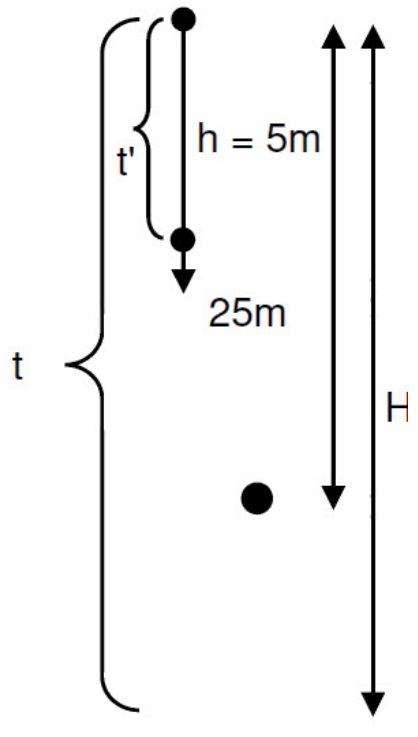
Solution:

For 1^{st} particle

$$H=\frac{1}{2}g(t)^2$$

$$5=\frac{1}{2}g(t')^2$$

$$t'=5\ sec$$



For 2^{nd} particle

$$H - 25 = \frac{1}{2}g(t-1)^2$$

$$\frac{1}{2}g(t)^2 - 25 = \frac{1}{2}g(t)^2 + \frac{g}{2} - gt$$

$$t = 3\ sec$$

$$H = \frac{1}{2} \times 10 \times 9 = 45\ m$$

Answer 2

Correct answers is B

Solution:

Horizontal component of velocity of bomb & fighter jet are same So, bomb will remains just below the jet, path is straight line w.r.t. pilot.

Answer 3

Correct answers is D

Solution:

$$y = \alpha x - \beta x^2, \qquad y = \alpha x\left(1 - \frac{x}{\alpha/\beta}\right)$$

Compare with $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$$\tan\theta = \alpha \implies \theta = \tan^{-1}\alpha$$

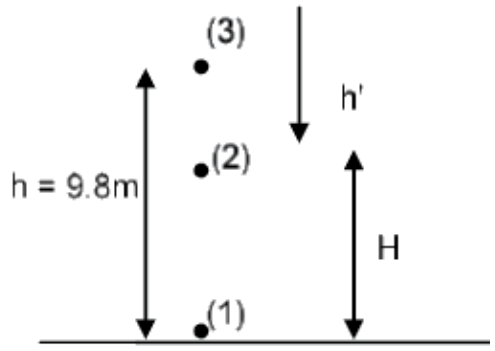
$$\frac{R}{H} = \frac{4}{\tan\theta}$$

$$H = \frac{R\tan\theta}{4} = \frac{\alpha^2}{4\beta}$$

Answer 4

Correct answers is D

Solution:



For first drop. $h = \frac{1}{2}g(2n)^2$

For second drop. $h' = \frac{1}{2}g(n)^2$

$$\frac{h}{h'} = \frac{4}{1}$$

$$h' = \frac{h}{4} = \frac{9.8}{4}$$

So height of second drop

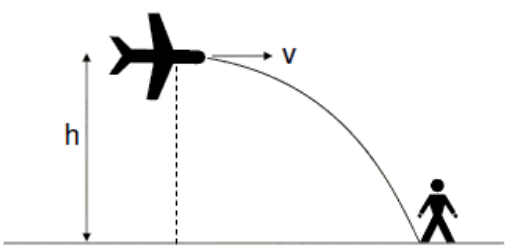
$$H = h - h' = 9.8 - \frac{9.8}{4}$$

$$= \frac{3}{4} \times 9.8 = 7.35\ m$$

Answer 5

Correct answers is D

Solution:



The aero plane imparts v velocity to the food packet horizontally when dropping.

Horizontal range of food packet $= v \times \sqrt{\frac{2h}{g}}$

Also. Horizontal distance. travelled by aero plane $= v \times \sqrt{\frac{2h}{g}}$

thus, vertical distance between aero plane and person $= h$

distance $= \sqrt{\frac{2v^2h}{g} + h^2}$

Answer 6

Correct answers is B

Solution:

We can determine the magnitude of $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$ either algebraically or graphically.

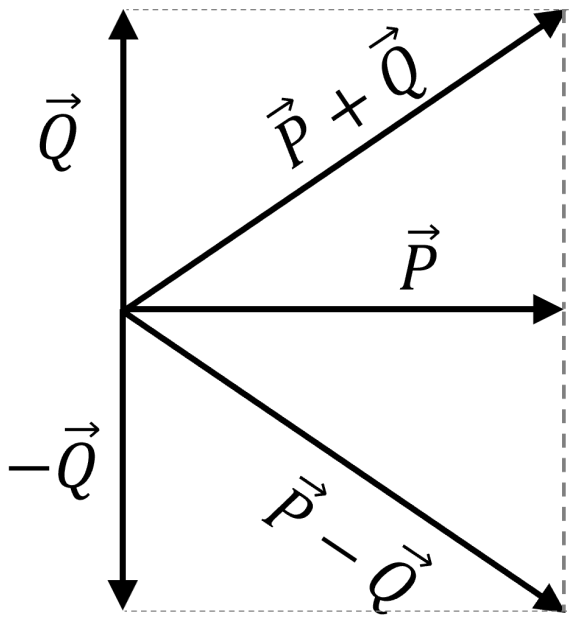
Algebraically:

$$|\vec{P} + \vec{Q}| = \sqrt{|\vec{P} + \vec{Q}|^2} = \sqrt{(\vec{P} + \vec{Q}).(\vec{P} + \vec{Q})} = \vec{P}.\vec{P} + \vec{Q}.\vec{Q} + 2\vec{P}.\vec{Q}$$

Now, $\overrightarrow{P} . \overrightarrow{Q} = PQ \cos 90 = 0$
So, $|\overrightarrow{P} + \overrightarrow{Q}| = \sqrt{P^2 + Q^2}$

Similarly, $|\overrightarrow{P} - \overrightarrow{Q}| = \sqrt{P^2 + Q^2}$

Now, let's derive the same graphically



$\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ at θ_1
$F_{\text{net 1}} = \sqrt{P^2 + Q^2 + P^2 + Q^2 + 2(P^2 + Q^2)\cos\theta_1}$
$F_{\text{net 2}} = \sqrt{P^2 + Q^2 + P^2 + Q^2 + 2(P^2 + Q^2)\cos\theta_2}$
If $F_{\text{net 1}} = \sqrt{3(P^2 + Q^2)}$
$= \sqrt{2(P^2 + Q^2) + 2(P^2 + Q^2)\cos\theta_1}$
$\implies \theta_1 = \dfrac{P^2 + Q^2}{2(P^2 + Q^2)}$
$\implies \theta_1 = 60^\circ$
$F_{\text{net 2}} = \sqrt{2(P^2 + Q^2)}$
$= \sqrt{2(P^2 + Q^2) + 2(P^2 + Q^2)\cos\theta_2}$
$\implies \cos\theta_2 = 0$
$\implies \theta_2 = 90^\circ$

Answer 7

Solution:
$v^2 = 2x + 20$
$\implies 2v\dfrac{dv}{dx} = 2$
$\implies a = \dfrac{2}{2} = 1\ m/s^2$

Answer 8

Correct answers is B

Solution:
$dx = vdt$
$x = \int_1^2 \alpha t dt + \int_1^2 \beta t^2 dt$

$$x = \left(\frac{\alpha t^2}{2}\right)_1^2 + \left(\frac{\beta t^3}{3}\right)_1^2;$$

$$x = \frac{3}{2}\alpha + \frac{7}{3}\beta$$

Answer 9

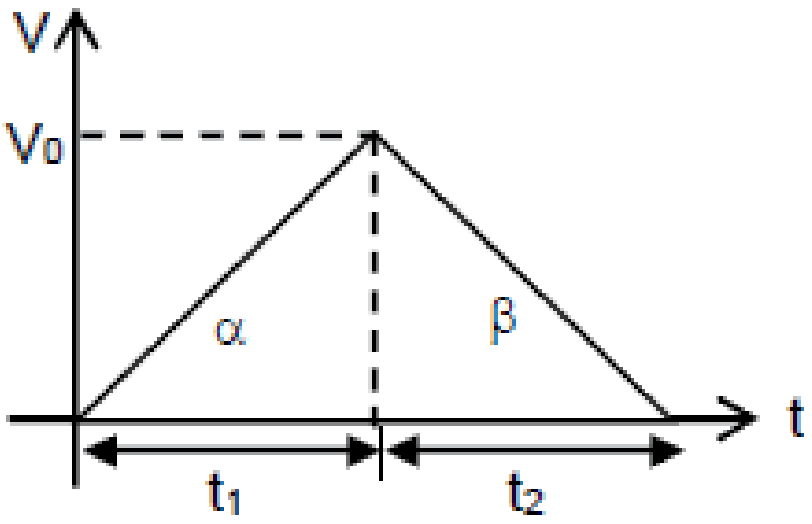
Correct answers is B

Solution:

$$V_0 = \alpha t_1 = \beta t_2.$$

$$t = t_1 + t_2 = \frac{V_0}{\alpha} + \frac{V_0}{\beta}$$

$$V_0 = \left(\frac{\alpha\beta}{\alpha+\beta}\right) t$$



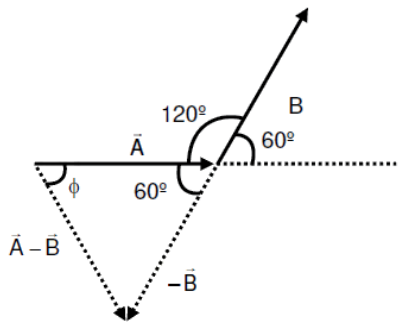
$$S_1 + S_2 = S = \text{Area of } V - t \text{ curve}$$

$$= \frac{1}{2}(t)V_0 = \frac{1}{2}(t)\left(\frac{\alpha\beta}{\alpha+\beta}\right)t = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

Answer 10

Correct answers is C

Solution:



$$\tan\phi = \frac{B\sin 60^\circ}{A - B\cos 60^\circ}$$

$$\phi = \tan^{-1}\left(\frac{B\sin 60^\circ}{A - B\cos 60^\circ}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}B}{2A - B}\right)$$

Answer 11

Correct answers is D

Solution:

$$\vec{a} = 8\hat{i} + 2\hat{j}$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

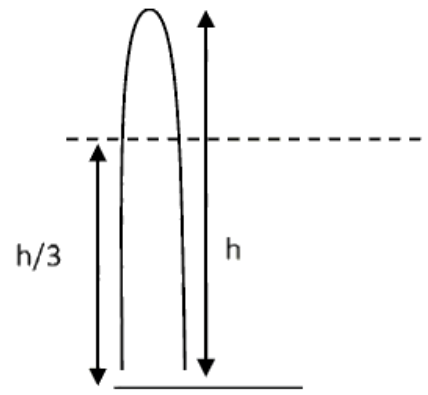
$$\vec{s} = \frac{1}{2}(8\hat{i} + 2\hat{j}) \times 100$$

$$\vec{s} = 400\hat{i} + 100\hat{j}$$

Answer 12

Correct answers is C

Solution:



$$V = \sqrt{2gh}$$

$$\frac{h}{3} = \sqrt{2ght} - \frac{1}{2}gt^2$$

$$gt^2 - 2\sqrt{2ght} + \frac{2h}{3} = 0$$

$$t = \frac{2\sqrt{2gh} \pm \sqrt{8gh - 4g \times \frac{2h}{3}}}{2g}$$

$$t = \frac{2\sqrt{2gh} \pm \sqrt{\frac{16gh}{3}}}{2g}$$

$$t = \frac{2\sqrt{2gh} \pm 4\sqrt{\frac{gh}{3}}}{2g}$$

$$\frac{t_1}{t_2} = \frac{2\sqrt{2gh} - 4\sqrt{\frac{gh}{3}}}{2\sqrt{2gh} + 4\sqrt{\frac{gh}{3}}}$$

$$\frac{t_1}{t_2} = \frac{2\sqrt{2} - \frac{4}{\sqrt{3}}}{2\sqrt{2} + \frac{4}{\sqrt{3}}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

Answer 13

Solution:

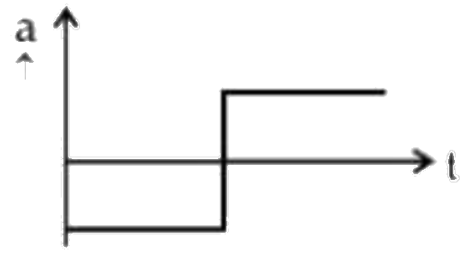
$$v^2 = u^2 + 2as$$
$$0 = 100 - 2 \times a \times 0.5$$
$$a = 100\ m/s^2$$

Retarding force $= ma = 0.1 \times 100 = 10\ N$

Answer 14

Correct answers is A

Solution:



Answer 15

Correct answers is C

Solution:

Acceleration of ball $a = \dfrac{F}{m}$

$$a = \frac{F_0}{m}\left[1 - \left(\frac{t-T}{T}\right)^2\right]$$

$$\frac{dv}{dt} = \frac{F_0}{m}\left[1 - \left(\frac{t-T}{T}\right)^2\right]$$

$$\int_0^v dv = \frac{F_0}{m}\int_0^{2T}\left[1 - \left(\frac{t-T}{T}\right)^2\right]dt$$

$$v = \frac{F_0}{m}\left[1 - \frac{1}{3T^2}(t-T)^3\right]_0^{2T}$$

$$v = \frac{F_0}{m}\left[\left(2T - \frac{1}{3T^2}(2T-T)^3\right) - \left(0 - \frac{(-T)^3}{3T^2}\right)\right]$$

$$v = \frac{4F_oT}{3M}$$

Answer 16

Correct answers is A

Solution:

$$\omega_1 = \frac{2\pi}{T_1} = 2\pi$$

$$\omega_2 = \frac{2\pi}{T_2} = \frac{2\pi}{8}$$

$$\implies \quad \frac{\omega_1}{\omega_2} = 8$$

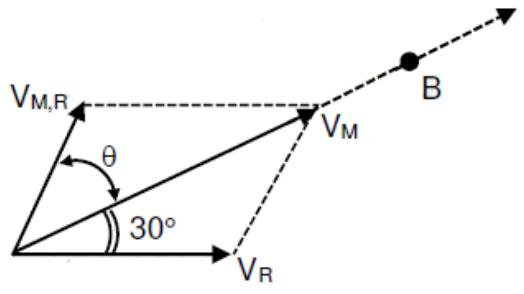
Answer 17

Solution:

$\overrightarrow{V}_M = \overrightarrow{V}_{M,R} + \overrightarrow{V}_R$

V_M should be along line AB

As $|\overrightarrow{V}_{M,R}| = |\overrightarrow{V}_R|$



$\therefore$ V_M should be along angle bisector of angle between $\overrightarrow{V}_{M,R}$ and $\overrightarrow{V}_R$

$\therefore$ $\theta = 30^\circ$

Answer 18

Correct answers is B

Solution:

$$v^2 = v_c^2 + 2al \qquad [\text{length of train } = 2l]$$
$$v_c^2 = u^2 + 2al$$
$$v^2 - v_c^2 = v_c^2 - u^2$$
$$v_c = \sqrt{\frac{v^2 + u^2}{2}}$$

Answer 19

Solution:

$S_1 = S_2$

$$35t + \frac{1}{2}(-g)t^2 = 35(t-3) + \frac{1}{2}(-g)(t-3)^2$$

$$35t - \frac{1}{2}gt^2 = 35t - 35 \times 3 - \frac{1}{2}g(t^2 - 6t + 9)$$

$$35 \times 3 + 45 = 30t$$

$$t = \frac{150}{30} = 5$$

height $h = 35 \times 5 - \frac{1}{2} \times 10 \times 5^2$

$h = 175 - 125 = 50\ m$

Answer 20

Correct answers is A

Solution:

Resultant $(\overrightarrow{R})$

$$= \hat{i}(10\cos 30^\circ + 20\cos 60^\circ - 15\cos 30^\circ - 15\cos 45^\circ + 20\cos 45^\circ)$$
$$+ \hat{j}(10\sin 30^\circ + 20\sin 60^\circ + 15\sin 30^\circ - 15\sin 45^\circ - 20\sin 45^\circ)$$
$$= 9.25\hat{i} + 5\hat{j}$$

Answer 21

Correct answers is C

Solution:

Suppose length of escalator $= L$

Speed of man w.r.t. escalator $= \dfrac{L}{t_1}$

Speed of escalator $= \dfrac{L}{t_2}$

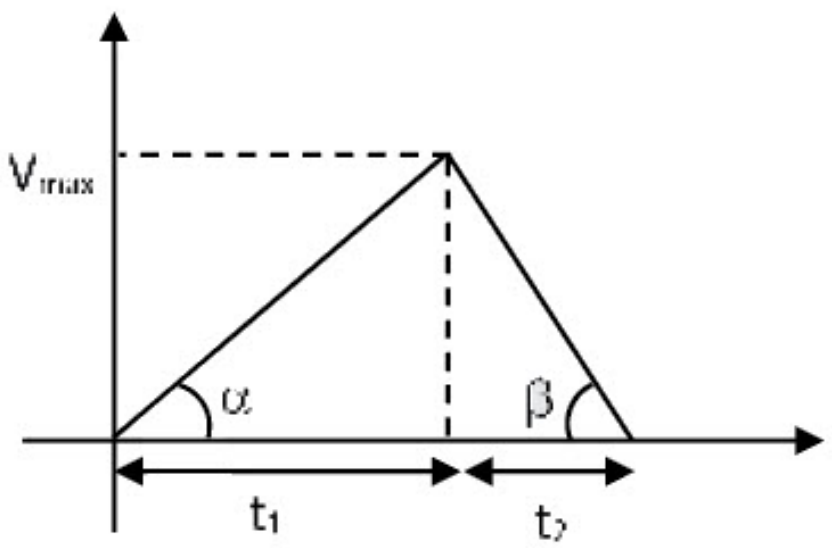
Speed of man w.r.t. ground when escalator is moving $= \dfrac{L}{t_1} + \dfrac{L}{t_2}$

Time taken by the man to walk on the moving escalator $= \dfrac{L}{\dfrac{L}{t_1} + \dfrac{L}{t_2}} = \dfrac{t_1 t_2}{t_1 + t_2}$

Answer 22

Correct answers is A

Solution:



$$\tan\alpha = \frac{V_{\max}}{t_1}\ ; \quad a_1 = \frac{V_{\max}}{t_1}$$

$$\tan\beta = \frac{V_{\max}}{t_2}\ ; \quad a_2 = \frac{V_{\max}}{t_2}$$

$$\frac{a_1}{a_2} = \frac{t_2}{t_1}\ ; \Longrightarrow \quad \frac{t_2}{t_1} = \frac{a_1}{a_2}$$

Answer 23

Solution:

$$a = \frac{vdv}{dx}$$

$$v = \sqrt{5000 + 24x} \times \frac{1}{2\sqrt{5000 + 24x}} \times 24$$

$$a = 12\ m/s^2$$

Answer 24

Correct answers is A

Solution:

$$\theta = 45^\circ$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(25)^2 \times \left(\frac{1}{2}\right)}{2 \times 10}$$

$$= \frac{125}{8}\, m$$

and time $t = \frac{T}{2}$

$$= \frac{u \sin\theta}{g} = \frac{25\left(\frac{1}{\sqrt{2}}\right)}{10}$$

$$= \frac{25}{10\sqrt{2}} = \frac{5}{2\sqrt{2}}\, s$$

Answer 25

Correct answers is B

Solution:

$$|\overrightarrow{X} - \overrightarrow{Y}| = n|\overrightarrow{X} + \overrightarrow{Y}|$$

$$|\overrightarrow{X}|^2 + |\overrightarrow{Y}|^2 - 2|\overrightarrow{X}||\overrightarrow{Y}|\cos\theta =$$

$$n^2\left[|\overrightarrow{X}|^2 + |\overrightarrow{Y}|^2 + 2|\overrightarrow{X}||\overrightarrow{Y}|\cos\theta\right]$$

As $|\overrightarrow{X}| = |\overrightarrow{Y}|$

$$2|\overrightarrow{X}|^2 - 2|\overrightarrow{X}|^2\cos\theta =$$

$$2n^2|\overrightarrow{X}|^2 + 2n^2|\overrightarrow{X}|^2\cos\theta$$

$$1 - \cos\theta = n^2 + n^2\cos\theta$$

$$\cos\theta = \frac{1 - n^2}{1 + n^2}$$

$$\theta = \cos^{-1}\left(\frac{n^2 - 1}{-n^2 - 1}\right)$$

Answer 26

Solution:

$$a = \frac{F}{m} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$$

$$\overrightarrow{r_f} - \overrightarrow{r_i} = \overrightarrow{u}t + \frac{1}{2}\overrightarrow{a}t^2$$

$$x\hat{i} + y\hat{j} + z\hat{k} = \frac{1}{2} \times \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}(4)^2$$

$$x\hat{i} + y\hat{j} + z\hat{k} = 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$$b = 12$$

Answer 27

Correct answers is C

Solution:

$t = mx^2 + nx$

Differentiating w.r.t. t

$1 = 2mxv + nv$

$1 = v(2mx + n)$

Again differentiating w.r.t. t

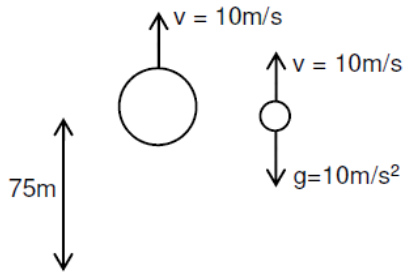
$\frac{dv}{dt} \times (2mx + n) + 2mv^2 = 0$;

$a = -2mv^3$

Answer 28

Correct answers is A

Solution:



For stone

$75 = -10t + \frac{1}{2}gt^2$

$75 = -10t + 5t^2$

$t^2 - 2t - 15 = 0$

$t = 5\ sec$

Height of balloon

$H = vt + 75$

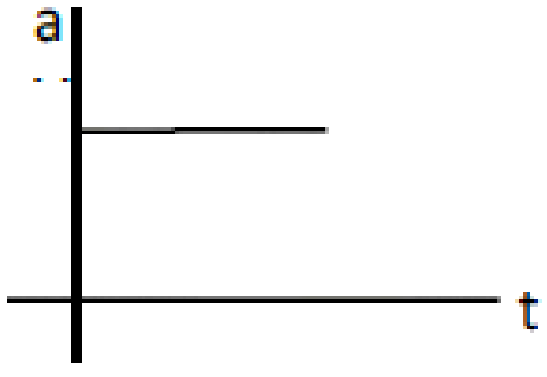
$H = 10 \times 5 + 75 = 125\ m.$

Answer 29

Correct answers is D

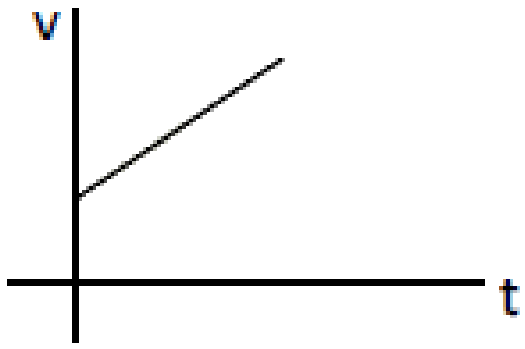
Solution:

$a =$ constant



$\frac{dv}{dt} =$ constant $= a$

$v = u + at$



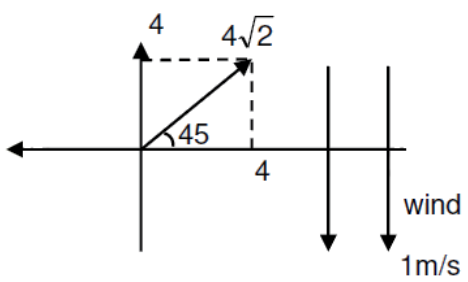
$$\frac{dx}{dt} = v = u + at$$

$$x = ut + \frac{1}{2}at^2$$

Answer 30

Correct answers is B

Solution:



$$\overrightarrow{D} = V_{F,G} \times T$$
$$= [4\hat{i} + 4\hat{j} + (-\hat{j})] \times 3s$$
$$|\overrightarrow{D}| = 15\ m$$

Answer 31

Solution:

By energy conservation

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\implies \quad v = x\sqrt{\frac{k}{m}}$$

$$v = 0.05 \times \sqrt{\frac{100}{0.1}} = 0.5\sqrt{10}\ m/s$$

Time of flight of ball $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}}\ sec$

Range of ball $s = ut$

$$d = 0.5\sqrt{10} \times \left(\frac{2}{\sqrt{10}}\right) = 1\ m$$

Answer 32

Correct answers is C

Solution:

$$|\overrightarrow{P}+\overrightarrow{Q}|=n|\overrightarrow{P}-\overrightarrow{Q}|$$
$$|\overrightarrow{P}|^2+|\overrightarrow{Q}|^2+2|\overrightarrow{P}||\overrightarrow{Q}|\cos\theta$$
$$=n^2(|\overrightarrow{P}|^2+|\overrightarrow{Q}|^2-2|\overrightarrow{P}||\overrightarrow{Q}|\cos\theta)$$
$$2+2\cos\theta=n^2(2-2\cos\theta)$$
$$\theta=\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

Answer 33

Correct answers is A

Solution:

Let next drop after $t\ sec$ distance travelled by 1^{st} drop in $4\ sec$, is $S_1=\frac{1}{2}at^2=78.4\ m$ (t should be less then $4\ sec$) distance travelled by succeeding drop in $4-t\ sec$

$$S_2=\frac{1}{2}a(4-t)^2$$
$$S_1-S_2=34.3$$
$$78.4-4.9(4-t)^2=34.3$$
$$(4-t)^2=9\,;4-t=3$$
$$t=1\ sec$$

Answer 34

Correct answers is D

Solution:

$$v=v_0+gt+Ft^2$$
$$\frac{dS}{dt}=v_0+gt+Ft^2$$
$$\int_0^S dS=\int_0^1(v_0+gt+Ft^2)dt$$
$$S=v_0+\frac{g}{2}+\frac{F}{3}$$

Answer 35

Correct answers is C

Solution:

$$S = h + (e^2 h + e^4 h + \ldots\ldots\ldots) \times 2$$

$$= h + 2e^2 h \frac{1}{1 - e^2} = h \left[1 + \frac{2e^2}{1 - e^2} \right] = h \left(\frac{1 + e^2}{1 - e^2} \right)$$

$$t = \sqrt{\frac{2h}{g}} + 2 \times \sqrt{\frac{2h}{g}} \left[1 + e + \ldots\ldots\ldots \right]$$

$$= \sqrt{\frac{2h}{g}} \left[1 + 2 \times \frac{1}{1 - e} \right] = \sqrt{\frac{2h}{g}} \left(\frac{1 + e}{1 - e} \right)$$

$$< V > = \frac{h}{\sqrt{\frac{2h}{g}}} \left(\frac{1 + e^2}{1 - e^2} \right) \times \left(\frac{1 - e}{1 + e} \right)$$

$$= \sqrt{\frac{gh}{2}} \frac{1 + e^2}{(1 + e)^2} = \sqrt{\frac{10 \times 5}{2}} \frac{1 + \left(\frac{81}{100} \right)}{\left(1 + \sqrt{\frac{81}{100}} \right)^2} \approx 2.5$$

Answer 36

Correct answers is C

Solution:

$a = v \left(\frac{dv}{dx} \right)$ (where dv/dx is negative)

v is decreasing

So, a will increase, hence correct option is (c).

Answer 37

Correct answers is B

Solution:

$\overrightarrow{OA} = R[\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}]$

$\overrightarrow{OB} = R[\cos 60^\circ \hat{i} + (-\sin 60^\circ) \hat{j}]$

$\overrightarrow{OC} = R[-\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}]$

$\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$

$$= R \left[\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \right] \hat{i}$$

$$+ R \left[\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \right] \hat{j}$$

angle with the x-axis

$$\tan \alpha = \frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}$$

$$= \frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}$$

$$\alpha = \tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}\right]$$

Answer 38

Solution:

$$\overrightarrow{P} \times \overrightarrow{Q} = \overrightarrow{Q} \times \overrightarrow{P}$$

$$\overrightarrow{P} \times \overrightarrow{Q} = -(\overrightarrow{P} \times \overrightarrow{Q})$$

$$2(\overrightarrow{P} \times \overrightarrow{Q}) = 0$$

$$PQ\sin\theta = 0$$

$$\theta = 180^\circ \qquad (0^\circ < \theta < 360^\circ)$$

Answer 39

Correct answers is B

Solution:

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ ...(i)

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ ...(ii)

$\overrightarrow{AD} = 2\overrightarrow{AO}$ ...(iii)

Adding (i), (ii) and (iii) we get

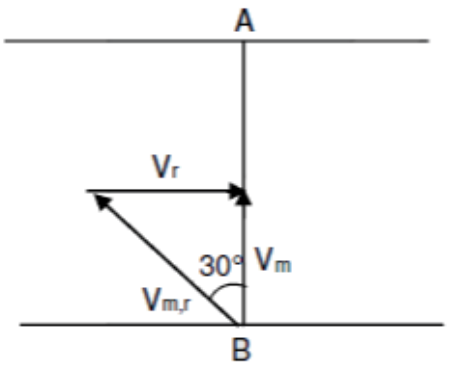
$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

And as per the polygon law of vector addition $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$

Both A and R are correct but R is not the correct explanation of A

Answer 40

Solution:



$$\sin 30^\circ = \frac{V_r}{V_{m,r}}\ ; \quad V_r = \frac{1}{2} \times 10$$

$$V_r = 5\ m/s$$

Answer 41

Correct answers is B

Solution:

(I) $\overrightarrow{A} + \overrightarrow{C} = \overrightarrow{B}$

(II) $\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C} = 0$

(III) $\overrightarrow{A} - \overrightarrow{B} - \overrightarrow{C} = 0$

(IV) $\overrightarrow{A} + \overrightarrow{B} - \overrightarrow{C} = 0$

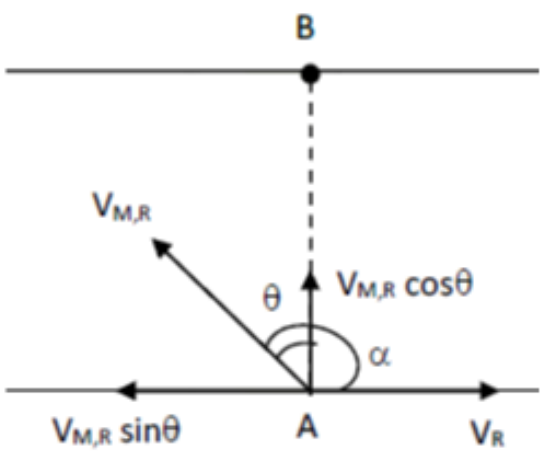
Answer 42

Solution:

$$V_R = 6\ km/hr$$

$$V_{M,R} = 12\ km/hr$$

V_M should be along AB



$$\therefore \quad V_R - V_{M,R}\sin\theta = 0$$

$$6 - 12\sin\theta = 0$$

$$\sin\theta = \frac{6}{12} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\alpha = 90 + \theta = 120^\circ$$

Answer 43

Correct answers is B

Solution:

$$\overrightarrow{A}\,.\,\overrightarrow{B} = |\overrightarrow{A} \times \overrightarrow{B}|$$

$$\implies \quad AB\cos\theta = AB\sin\theta$$

$$\therefore \quad \theta = 45^\circ$$

$$\therefore \quad |\overrightarrow{A} - \overrightarrow{B}| = \sqrt{A^2 + B^2 - 2AB\cos 45^\circ}$$

$$= \sqrt{A^2 + B^2 - \sqrt{2}AB}$$

Answer 44

Correct answers is D

Solution:

$$\overrightarrow{V} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$$

$$\overrightarrow{V}_{\text{at } t=2} = 2\hat{i} + 6\hat{j} + 9\hat{k}$$

angle with x-axis $\cos^{-1}\dfrac{2}{11} = \tan^{-1}\dfrac{\sqrt{117}}{2}$

angle with y-axis $\cos^{-1}\dfrac{6}{11} = \tan^{-1}\dfrac{\sqrt{85}}{6}$

None of the option is matching.

Answer 45

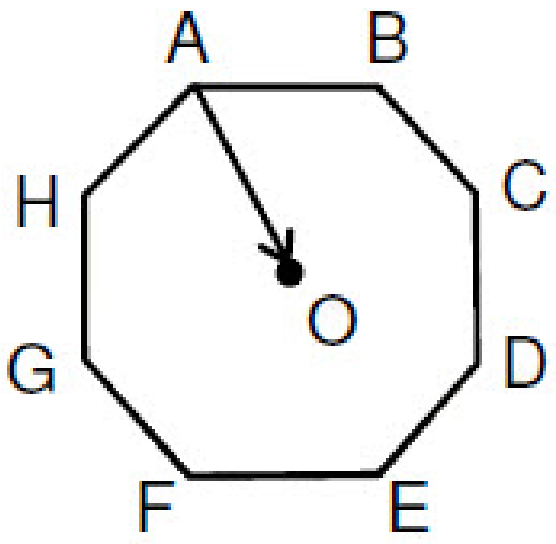
Correct answers is B

Solution:

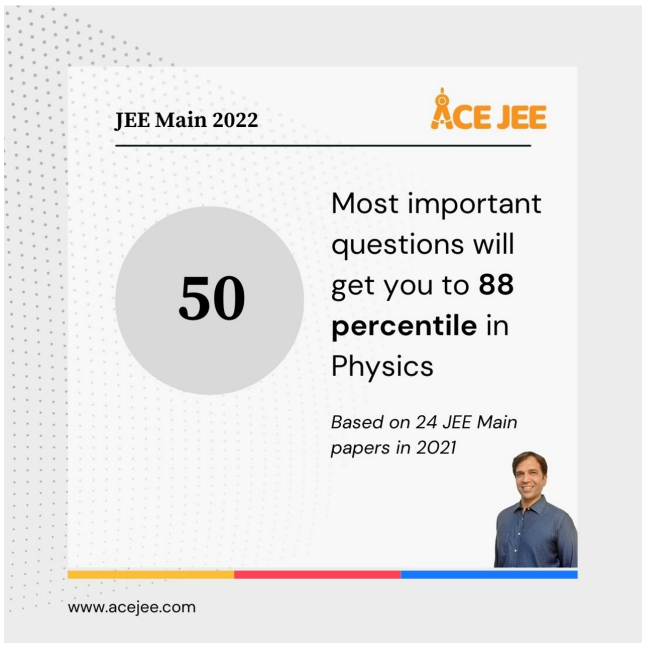
$$\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

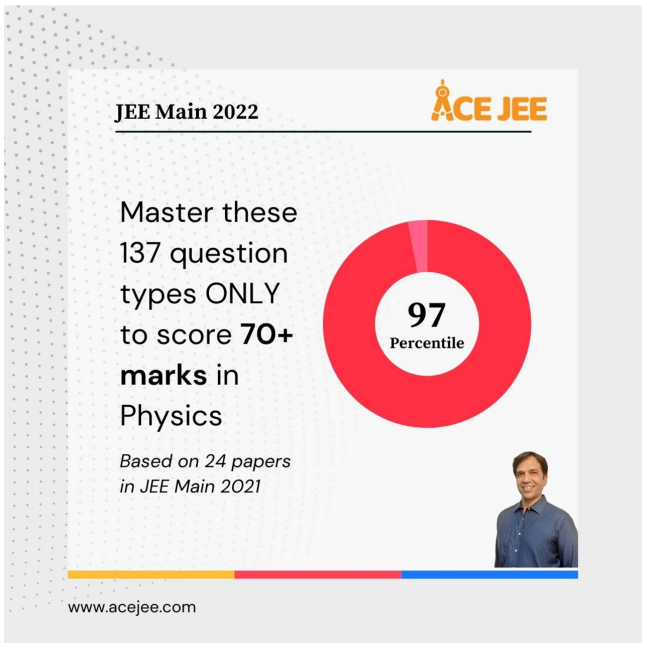
$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$$

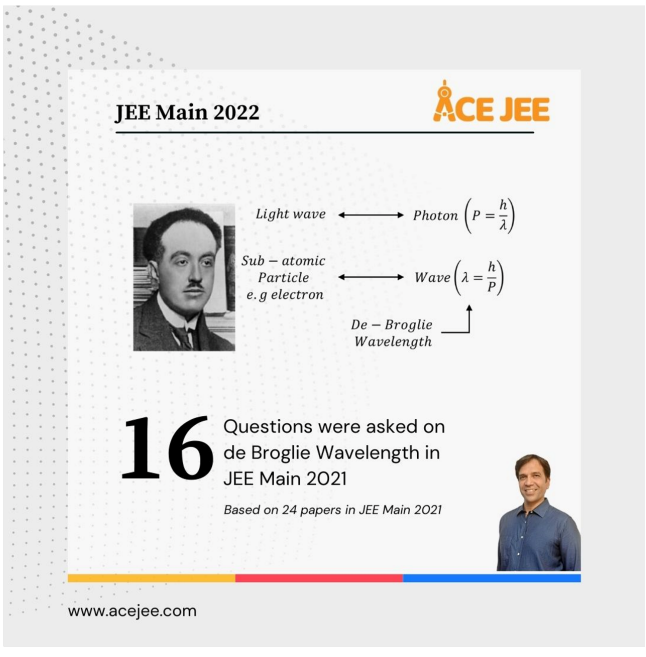
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$



$$\begin{aligned}
\implies \quad & \overrightarrow{AB} + (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}) + (\overrightarrow{AB} + \\
& \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}) + (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF}) + \\
& (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FG}) + (\overrightarrow{AB} + \overrightarrow{BC} + \\
& \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH}) \\
& [\overrightarrow{EF} = -\overrightarrow{AB},\ \overrightarrow{FG} = -\overrightarrow{BC},\ \overrightarrow{GH} = -\overrightarrow{CD}, \\
& \overrightarrow{HA} = -\overrightarrow{DE}] \\
\implies \quad & \overrightarrow{AB} + (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}) + (\overrightarrow{AB} + \\
& \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}) + (\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}) + (\overrightarrow{CD} + \\
& \overrightarrow{DE}) + (\overrightarrow{DE}) \\
\implies \quad & 4 \times (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}) = 4 \times \overrightarrow{AE} = 4 \times 2\overrightarrow{AO} \\
\implies \quad & 8(2\hat{i} + 3\hat{j} - 4\hat{k}) = 16\hat{i} + 24\hat{j} - 32\hat{k}
\end{aligned}$$







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