JEE Main 2021 | Center of Mass and Linear Momentum

Question 1

(Only one correct answer) 2021

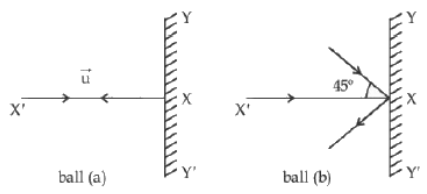
If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be

- (a) 400 %
- (b) 100 %
- (c) 300 %
- (d) 200 %

Question 2

(Only one correct answer) 2021

Two billiard balls of equal mass $30\ g$ strike a right wall with same speed of $108\ kmph$ (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball $'a'$ and ball $'b'$. by the wall along $'X'$ direction is :

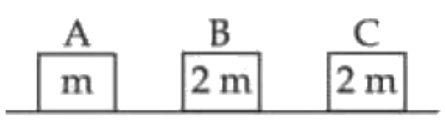


- (a) $\sqrt{2} : 1$
- (b) $1 : \sqrt{2}$
- (c) $2 : 1$
- (d) $1 : 1$

Question 3

(Only one correct answer) 2021

Three objects $A,\ B$ and C are kept in a straight line on a frictionless horizontal surface. The masses of $A,\ B$ and C are $m,\ 2m$ and $2m$ respectively. A moves towards B with a speed of $9\ m/s$ and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is :



(a) $3\ m/s$

(b) $9\ m/s$

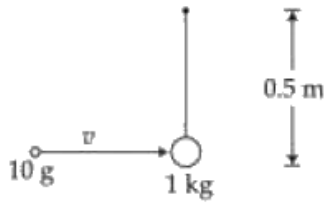
(c) $4\ m/s$

(d) $6\ m/s$

Question 4

(Integer type question) 2021

A bullet of $10\ g$, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity $100\ m/s$. The length of the pendulum is $0.5\ m$ and mass of the bob is $1\ kg$. The minimum value of $v = \ldots\ldots\ m/s$ so that the pendulum describes a circle. (Assume the string to be inextensible and $g = 10\ m/s^2$)



Question 5

(Only one correct answer) 2021

A body of mass M moving at speed V_0 collides elastically with a mass $'m'$ at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

(a) 1

(b) 4

(c) 2

(d) 3

Question 6

(Only one correct answer) 2021

A block moving horizontally on a smooth surface with a speed of $40\ m/s$ splits into two parts with masses in the ratio of $1:2$. If the smaller part moves at $60\ m/s$ in the same direction, then the fractional change in kinetic energy is :

(a) $\frac{1}{8}$

(b) $\frac{2}{3}$

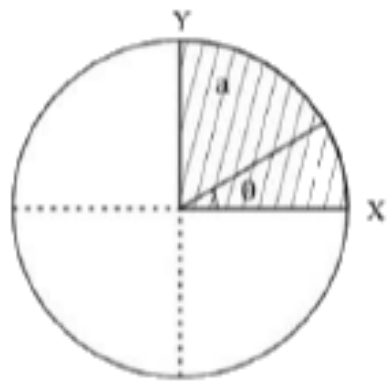
(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

Question 7

(Integer type question) 2021

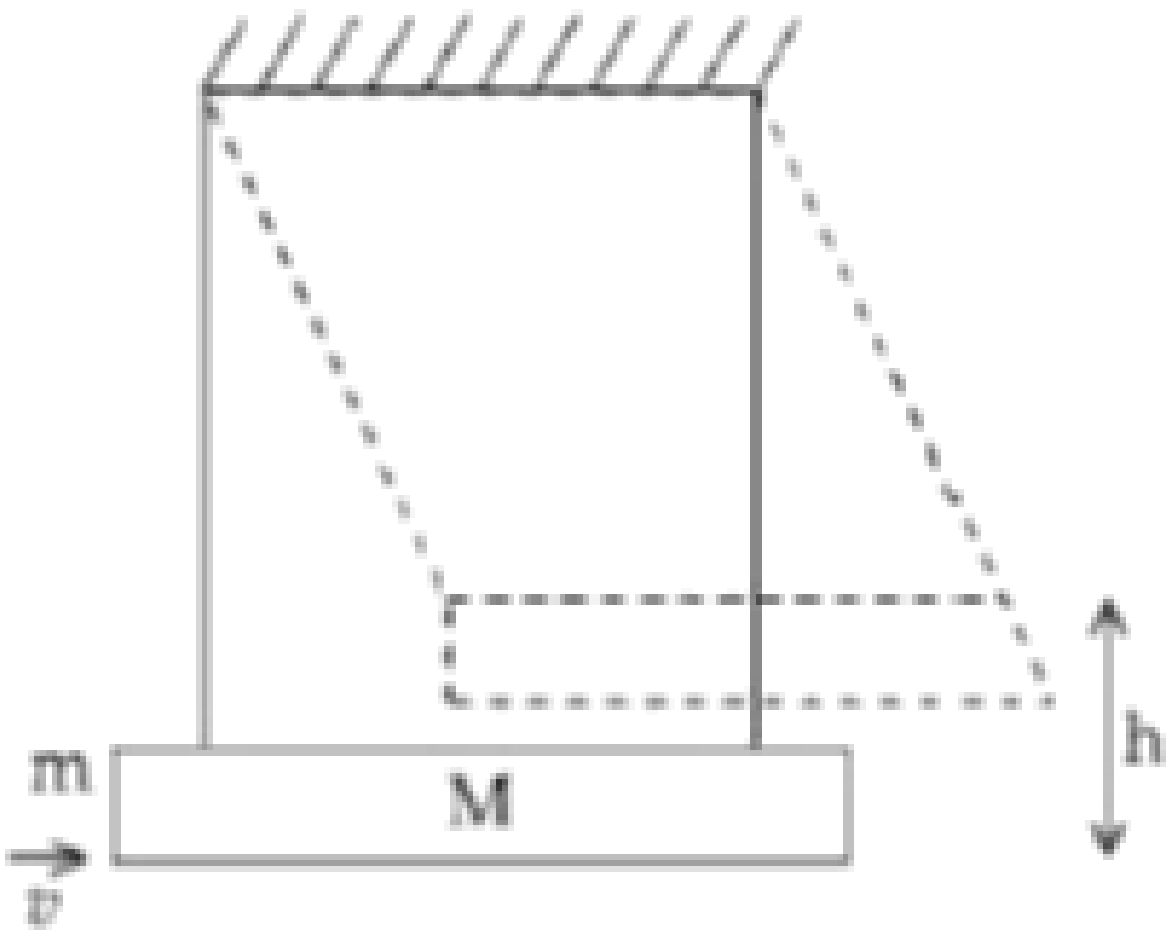
The disc of mass M with uniform surface mass density σ is shown in the figure. The center of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is ..... (Round off to the Nearest Integer) [a is an area as shown in the figure]



Question 8

(Only one correct answer) 2021

A large block of wood of mass $M = 5.99\ kg$ is hanging from two long massless cords. A bullet of mass $m = 10\ g$ is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their center of mass rising a vertical distance $h = 9.8\ cm$ before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is : (take $g = 9.8\ ms^{-2}$)



(a) $841.4\ m/s$

(b) $831.4\ m/s$

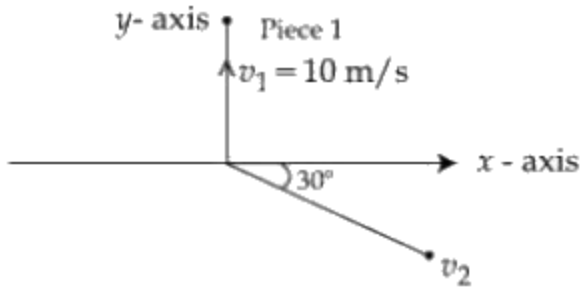
(c) $821.4\ m/s$

(d) $811.4\ m/s$

Question 9

(Integer type question) 2021

A ball of mass $10\ kg$ moving with a velocity $10\sqrt{3}\ m/s$ along the x-axis, hits another ball of mass $20\ kg$ which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of $10\ m/s$. The second piece starts moving at an angle of 30° with respect to the x-axis. The velocity of the ball moving at 30° with x-axis is $x\ m/s$. The configuration of pieces after collision is shown in the figure below. The vale of x to the nearest integer is .....



Question 10

(Integer type question) 2021

The projectile motion of a particle of mass $5\ g$ is shown in the figure.

The initial velocity of the particle is $5\sqrt{2}\ ms^{-1}$ and the resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}\ kgms^{-1}$. The value of x, to the nearest integer, is .....

Question 11

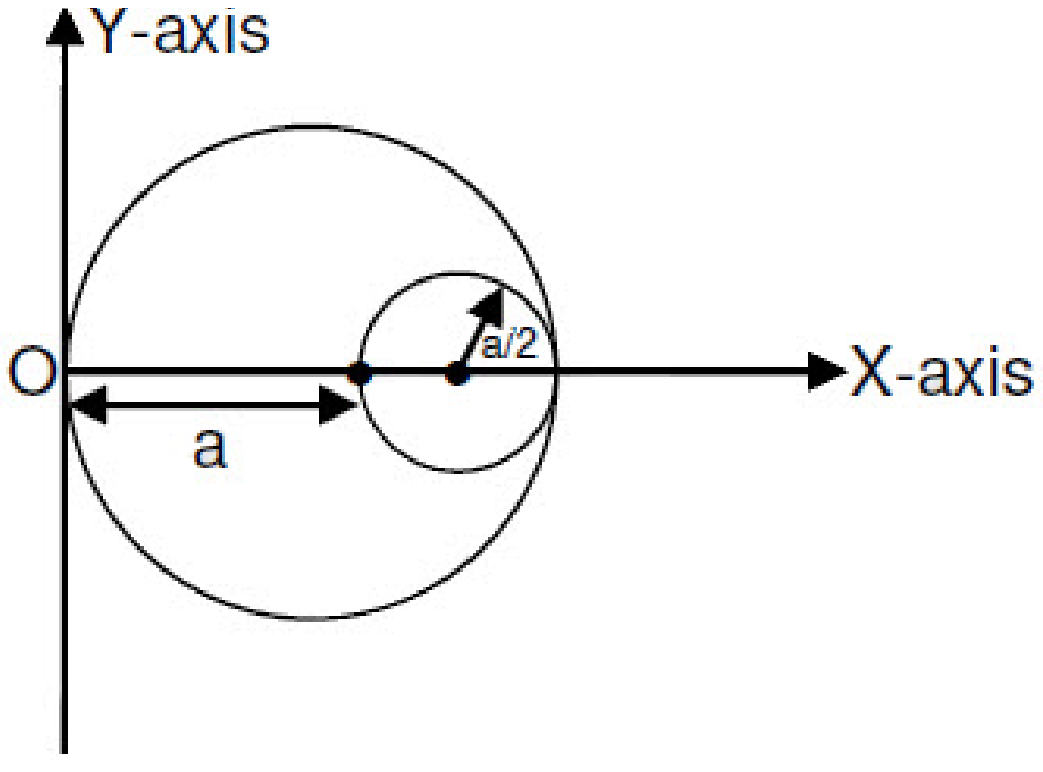
(Integer type question) 2021

A body of mass $2\ kg$ moving with a speed of $4\ m/s$, makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body center of mass is $\frac{x}{10}\ m/s$. Then the value of x is .....

Question 12

(Only one correct answer) 2021

A circular hole of radius $\left(\frac{a}{2}\right)$ is cut of a circular disc of radius $'a'$ as shown in figure. The centroid of the remaining circular portion with respect to point O will be



- (a) $\frac{10}{11}a$
- (b) $\frac{2}{3}a$
- (c) $\frac{5}{6}a$
- (d) $\frac{1}{6}a$

Question 13

(Integer type question) 2021

A block moving horizontally on a smooth surface with a speed of $40\ ms^{-1}$ splits into two equal parts. If one of the parts moves at $60\ ms^{-1}$ in the same direction, them the fractional change in the kinetic energy will be $x : 4$ where $x =$ .....

Question 14

(Integer type question) 2021

Two particles having masses $4\ g$ and $16\ g$ respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is $n : 2$. The value of n will be ..... .

Question 15

(Only one correct answer) 2021

An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal seeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

- (a) $1 : 1$
- (b) $1 : 2$
- (c) $2 : 1$

(d) $3 : 1$

Question 16

(Only one correct answer) 2021

The initial mass of a rocket is $1000\ kg$. Calculate at what rate the fuel should be burnt so that rocket is given an acceleration of $20\ ms^{-2}$. The gases come out at a relative speed of $500\ ms^{-1}$ with respect to the rocket: [Use $g = 10\ m/s^2$]

(a) $6.0 \times 10^2\ kg\ s^{-1}$

(b) $500\ kg\ s^{-1}$

(c) $10\ kg\ s^{-1}$

(d) $60\ kg\ s^{-1}$

Question 17

(Integer type question) 2021

A ball with a speed of $9\ m/s$ collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is .....

Question 18

(Integer type question) 2021

Two solids A and B of mass $1\ kg$ and $2\ kg$ respectively are moving with equal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$, so the value of A will be .....

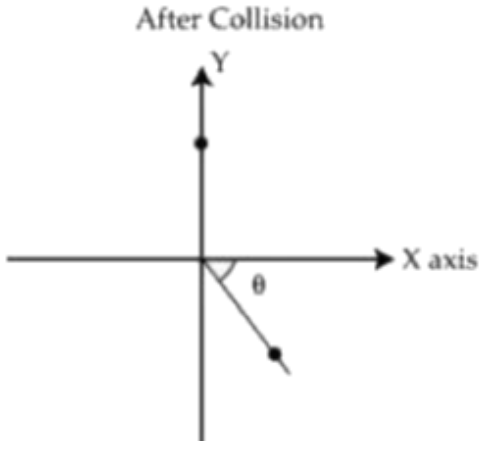
Question 19

(Integer type question) 2021

A ball of mass $10\ kg$, moving with a velocity $10\sqrt{3}\ ms^{-1}$ along X-axis, hits another ball of mass $20\ kg$ which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of $10\ m/s$. The second piece starts moving at a speed of $20\ m/s$ at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure.

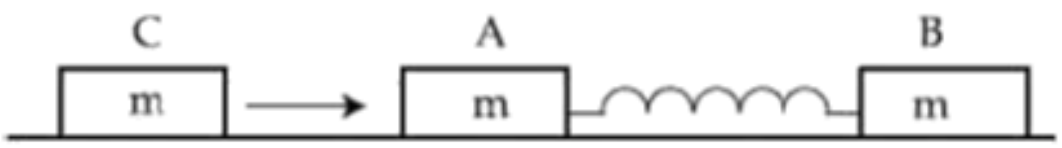
The value of θ to the nearest integer is .....



Question 20

(Only one correct answer) 2021

Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is



- (a) $v\sqrt{\frac{m}{2K}}$
- (b) $\sqrt{\frac{mv}{K}}$
- (c) $\sqrt{\frac{mv}{2K}}$
- (d) $\sqrt{\frac{m}{2K}}$

Question 21

(Only one correct answer) 2021

Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m << M$, body Q will have a maximum speed equal to $2u$ after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) A is correct but R is not correct.
- (b) Both A and R are correct but R is not the correct explanation of A.
- (c) A is not correct but R is correct.
- (d) Both A and R are correct and R is the correct explanation of A.

Answer 1

Correct answers is B

Solution:

$$K.E. \implies K = \frac{P^2}{2m}$$

$$P \propto \sqrt{K}$$

$$\frac{P_2}{P_1} = \sqrt{\frac{K_2}{K_1}}$$

$$\implies \frac{P_2}{P_1} = \sqrt{\frac{4K}{K}}$$

$$\implies \frac{P_2}{P_1} = 2$$

$$\implies \frac{P_2 - P_1}{P_1}\,\% = \left(\frac{P_2}{P_1} - 1\right) \times 100$$

$$= (2-1) \times 100$$

$$\implies \frac{\Delta P}{P_1}\,\% = 100\,\%$$

Answer 2

Correct answers is A

Solution:

ratio of impulse

$$\frac{J_1}{J_2} = \frac{\Delta P_1}{\Delta P_2}$$

$$= \frac{m(u+u)}{m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)} = \sqrt{2} : 1$$

Answer 3

Correct answers is A

Solution:

$9m = mv_A + 2mv_B$

$9 = v_A + 2v_B$ ...(i)

$9 = v_B - v_A$ ...(ii)

Thus $v_B = 6\ m/s$

Collision between B & C

$2mv_B = 4mv_C$

$v_C = 3\ m/s$

Answer 4

Solution:

Final velocity of system (bullet + mass) should be $= \sqrt{5gR}$

Now $M\sqrt{5gR} = m(v + 100)$

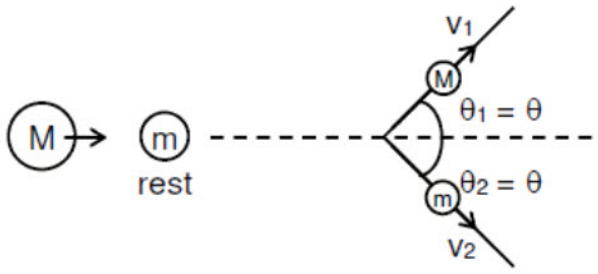
thus $v = \frac{M}{m}\sqrt{5}gr - 100$

$v = 500 - 100 = 400\ m/s$

Answer 5

Correct answers is D

Solution:



During collision, linear momentum is conserved along x and y directions

Along y-direction

$mv_2 \sin\theta = Mv_1 \sin\theta$

$mv_2 = Mv_1 \cdots$(i)

Along x-direction

$Mv_0 = mv_2 \cos\theta + Mv_1 \cos\theta \cdots$(ii)

From (i) and (ii)

$Mv_0 = Mv_1 \cos\theta + Mv_1 \cos\theta$

$\implies v_1 = \dfrac{v_0}{2\cos\theta} \cdots$(iii)

and $v_2 = \dfrac{M}{m}\dfrac{v_0}{2\cos\theta} \cdots$(iv)

Now, since the collision is elastic, mechanical energy i.e. kinetic energy is conserved as well, i.e.

$\dfrac{1}{2}Mv_0^2 = \dfrac{1}{2}Mv_1^2 + \dfrac{1}{2}mv_2^2$

or $Mv_0^2 = Mv_1^2 + mv_2^2$

Substituting values for v_1 and v_2 from (iii) and (iv), we get

$\dfrac{M}{m} = 4\cos^2\theta - 1$

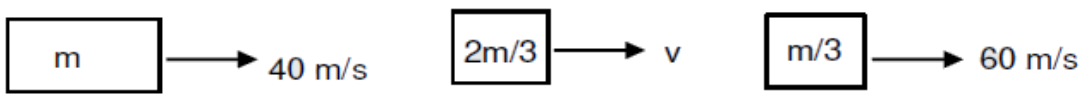
Now, maximum value of $\cos\theta = 1$

so, $\left(\dfrac{M}{m}\right)_{\max} = 4 - 1 = 3$

Answer 6

Correct answers is A

Solution:



$m \times 40 = \dfrac{2}{3}mv + \dfrac{m}{3} \times 60$

$120 = 2v + 60$

$v = 30\ m/s$

Now $K_i = \dfrac{1}{2} \times m \times 40^2 = 800\ m$

$K_f = \dfrac{1}{2} \times \dfrac{2m}{3}(30)^2 + \dfrac{1}{2} \times \dfrac{m}{3}(60)^2$

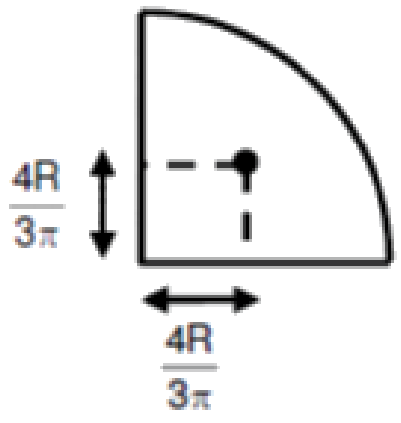
$= \dfrac{1}{6}[m(1800 + 3600)]$

$$= \frac{m(5400)}{6} = 900\ m$$

$$\frac{\Delta KE}{K_i} = \frac{900\ m - 800\ m}{800\ m} = \frac{1}{8}$$

Answer 7

Solution:



So, value of x is 4

Answer 8

Correct answers is B

Solution:

$$mu = (m+M)v$$
$$0.01u = (0.01 + 5.99)v$$
$$u = 600\ v \qquad \ldots(\text{i})$$

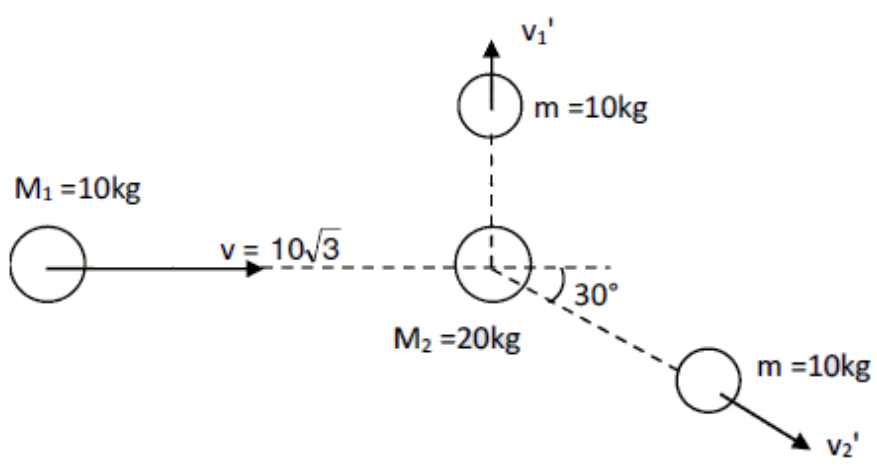
$$\frac{1}{2}(m+M)v^2 = (m+M)gh$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 9.8 \times 10^{-2}} = 1.386 \qquad \ldots(\text{ii})$$

From equation (i) and (ii), $u = 831.4\ m/s$

Answer 9

Solution:



Applied conservation of momentum along horizontal axis

$$M_1 v_1 = m v_2' \cos 30^\circ$$

$$v_2' = \frac{10 \times 10\sqrt{3}}{10 \cos 30^\circ} = 20\ m/s$$

Answer 10

Solution:

$$J = \Delta P = Ft$$

$$= mgT_f = 5 \times 10^{-3} \times 10 \times \frac{2 \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}}{g}$$

$$= 5 \times 10^{-3} \times 10 = 0.05\ kg - m/s$$

$$= 5 \times 10^{-2}\ kg - m/s$$

$$x = 5$$

Answer 11

Solution:

From linear momentum conservation

$2 \times 4 + 0 = 2 \times 1 + m_2 v_2$

From the definition of elastic collision

$v_2 - v_1 = e(u_1 - u_2)$

$v_2 - 1 = 1(4 - 0)$

$v_2 = 5$

$8 = 2 + m_2 \times 5$

$m_2 = 6/5$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{2 \times 4 + 0}{2 + \frac{6}{5}}$$

$$= 2.5\ m/s$$

$$x = 25$$

Answer 12

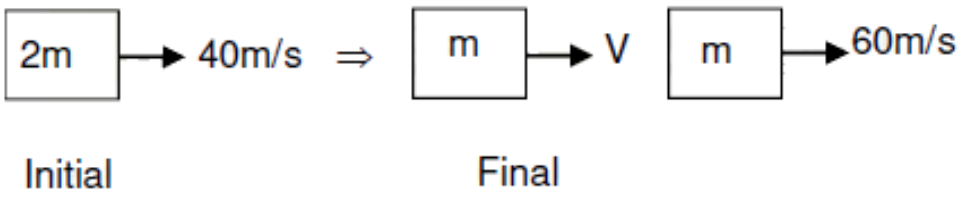
Correct answers is C

Solution:

$$X_{COM} = \frac{M(a) + \left(-\frac{M}{4}\right)\left(\frac{3}{2}a\right)}{M - \frac{M}{4}} = \frac{5a}{6}$$

Answer 13

Solution:



$2m \times 40 = 60 \times m + mv$

$v = 20\ m/s$

Now $\Delta KE = KE_f - KE_i$

$$= \frac{1}{2}m \times (20)^2 + \frac{1}{2}m \times (60)^2 - \frac{1}{2}2m \times (40)^2$$

$$= \frac{1}{2}m(800)$$

$$\implies KE_i = \frac{1}{2}2m \times (40)^2 = \frac{1}{2}m \times 3200$$

$$\text{Fractional change} = \frac{\Delta KE}{KE_i} = \frac{1}{4}$$

Answer 14

Solution:

$$P = \sqrt{2m(K.E.)}$$

$$\text{Thus,} \quad \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = 1:2$$

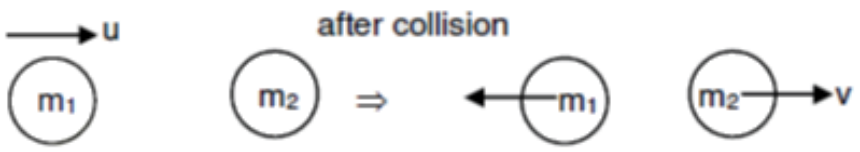
Answer 15

Correct answers is D

Solution:

Conservation of momentum

$$m_1 u = -m_1 v + m_2 v$$



Consider collision is elastic

$$e = 2v/u$$

$$u = 2v$$

$$m_1(2v) = -m_1 v + m_2 v$$

$$3m_1 = m_2$$

$$\frac{m_2}{m_1} = 3$$

Answer 16

Correct answers is D

Solution:

$$f_T - mg = ma$$

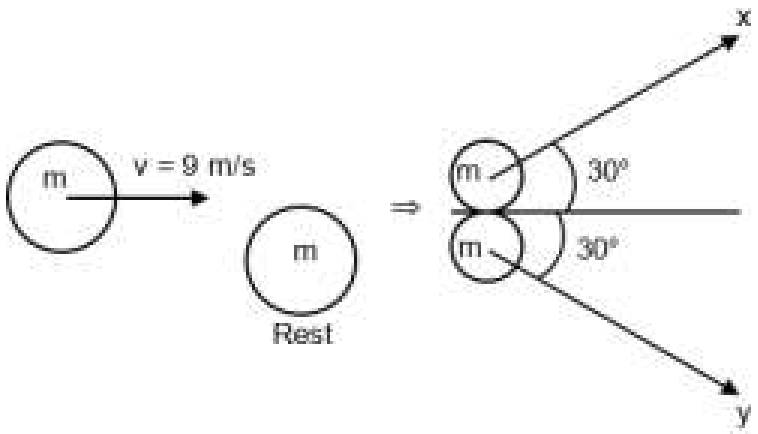
$$\frac{dm}{dt}V_r = mg + ma$$

$$\frac{dm}{dt}500 = 1000(10 + 20)$$

$$\frac{dm}{dt} = 60\ kg\ s^{-1}$$

Answer 17

Solution:



By momentum conservation along perpendicular direction

$$x \sin 30^\circ = y \sin 30^\circ$$

So, $$\frac{x}{y} = 1$$

Answer 18

Solution:

$$\frac{KE_A}{KE_B} = \frac{\frac{P_A^2}{2m_A}}{\frac{P_B^2}{2m_B}} = \frac{m_B}{m_A} = \frac{2}{1} \quad \Longrightarrow \quad A = 2$$

Answer 19

Solution:

From conservation of momentum along X axis :

$$\overrightarrow{P_i} = \overrightarrow{P_f}$$

$$10 \times 10\sqrt{3} = 200 \cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$

Answer 20

Correct answers is A

Solution:

In elastic collision of same mass particles interchange their velocity. So block 1 will come in rest and block 2 move with same velocity v, just after the collision. At maximum compression block 2 and 3 will have common velocity $\frac{v}{2}$ (from linear momentum conservation)

From energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}Kx^2$$

$$\frac{1}{2}mv^2 = Kx^2$$

$$x = \sqrt{\frac{mv^2}{2K}}$$

Answer 21

Correct answers is D

Solution: