JEE Main 2021 | Rotation

Question 1

(Only one correct answer) 2021

Two discs have moments of inertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the center. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by:

- (a) $\frac{I_1 I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$
- (b) $\frac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$
- (c) $\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$
- (d) $\frac{(I_1 - I_2)^2 \omega_1 \omega_2}{2(I_1 + I_2)}$

Question 2

(Integer type question) 2021

A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass $'m'$ travelling along the surface hits at one end of the rod with a velocity $'u'$ in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of $'x'$ will be .....

Question 3

(Only one correct answer) 2021

A system consists of two identical spheres each of mass $1.5\ kg$ and radius $50\ cm$ at the ends of a light rod. The distance between the centers of the two spheres is $5\ m$. What will be the moment of inertia of the system about an axis perpendicular to the line joining their centers and passing through its midpoint ?

- (a) $1.905 \times 10^5\ kg\ m^2$
- (b) $1.905\ kg\ m^2$
- (c) $18.75\ kg\ m^2$
- (d) $18.75 \times 10^5\ kg\ m^2$

Question 4

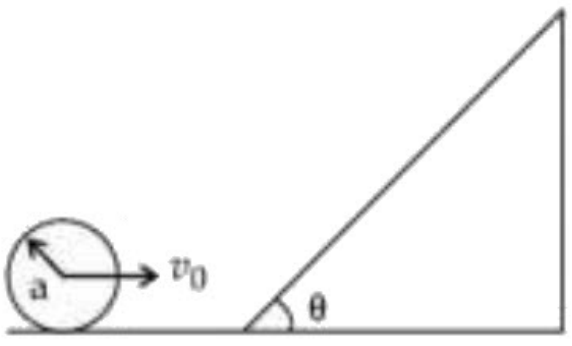
(Integer type question) 2021

A circular disc reaches from top to bottom of an inclined plane of length $'L'$. When it slips down the plane, it takes time $'t_1'$. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. Then value of x will be .....

Question 5

(Only one correct answer) 2021

A sphere of radius $'a'$ and mass $'m'$ rolls along a horizontal plane with constant speed V_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel ?



- (a) $\frac{10V_0^2}{7g\sin\theta}$
- (b) $\frac{V_0^2}{2g\sin\theta}$
- (c) $\frac{V_0^2}{5g\sin\theta}$
- (d) $\frac{2V_0^2}{5g\sin\theta}$

Question 6

(Only one correct answer) 2021

Angular momentum of a single particle moving with constant speed along circular path

- (a) Changes in magnitude but remains same in the direction
- (b) is zero
- (c) remains same in magnitude and direction
- (d) remains same in magnitude but changes in the direction

Question 7

(Integer type question) 2021

Two bodies, a ring and a solid cylinder of same material are rolling down without slipping and

inclined plane The radii of the bodies are same. The ratio of velocity of the center of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is .....

Question 8

(Only one correct answer) 2021

A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance $'h'$, the square of angular velocity of wheel will be:

- (a) $\frac{2gh}{I+mr^2}$
- (b) $\frac{2mgh}{I+2mr^2}$
- (c) $\frac{2mgh}{I+mr^2}$
- (d) $2gh$

Question 9

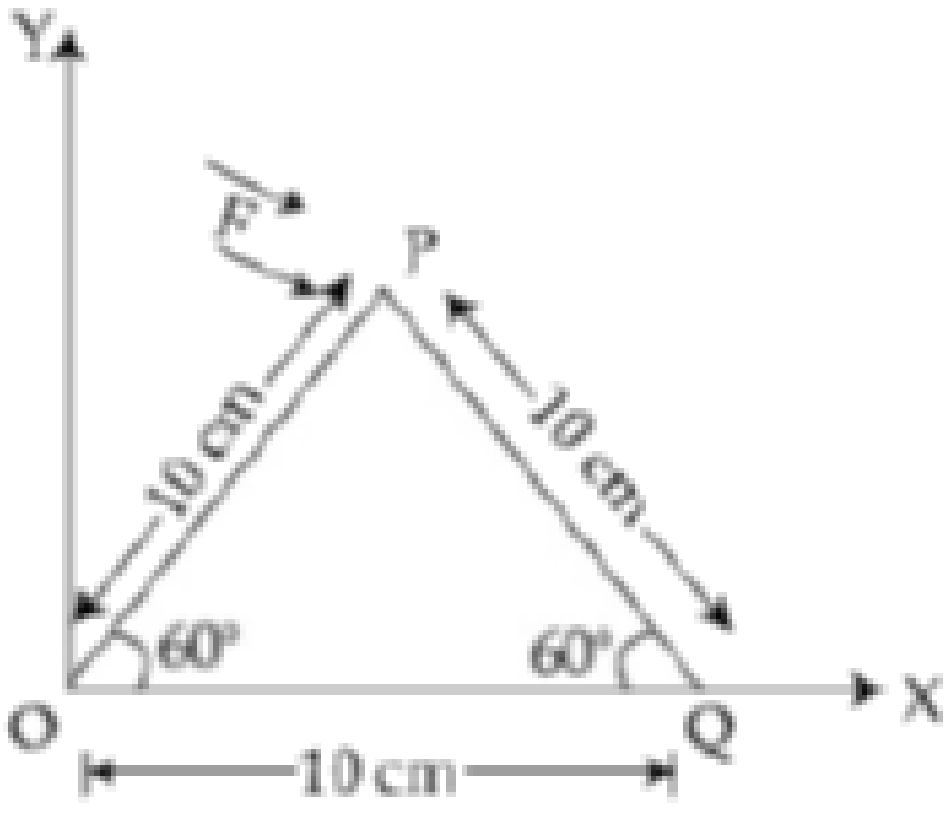
(Integer type question) 2021

A solid disc of radius $20\ cm$ and mass $10\ kg$ is rotating with an angular velocity of $600\ rpm$, about an axis normal to its circular plane and passing through its center of mass. The retarding torque required to bring the disc at rest in $10\ s$ is $\ldots\ldots \pi \times 10^{-1}\ Nm$.

Question 10

(Only one correct answer) 2021

A triangular plate is shown. A force $\overrightarrow{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque due to $\overrightarrow{F}$ about points O and Q are



- (a) $15 - 20\sqrt{3},\ 15 + 20\sqrt{3}$
- (b) $-15 + 20\sqrt{3},\ 15 - 20\sqrt{3}$
- (c) $-15 - 20\sqrt{3},\ 15 - 20\sqrt{3}$
- (d) $-15 + 20\sqrt{3},\ 15 + 20\sqrt{3}$

Question 11

(Only one correct answer) 2021

A huge circular arc of length $4.4\ ly$ subtends an angle $4s$ at the center of circle. How long it would take for a body to complete 4 revolution if its speed is $8\ AU$ per second ? Given :

$1\ ly = 9.46 \times 10^{15}\ m$ and $1\ AU = 1.5 \times 10^{11}\ m$

- (a) $4.1 \times 10^{8}\ s$
- (b) $4.5 \times 10^{10}\ s$
- (c) $7.2 \times 10^{8}\ s$
- (d) $3.5 \times 10^{6}\ s$

Question 12

(Only one correct answer) 2021

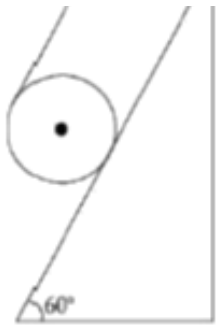
A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become :

- (a) $\omega \dfrac{M}{M + m}$
- (b) $\omega \dfrac{M}{M + 2m}$
- (c) $\omega \dfrac{M - 2m}{M + 2m}$
- (d) $\omega \dfrac{M + 2m}{M}$

Question 13

(Only one correct answer) 2021

A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :



[The coefficient of static friction, μ_s, is 0.4]

- (a) $\frac{mg}{5}$
- (b) 0
- (c) $\frac{7}{2}\ mg$
- (d) $5\ mg$

Question 14

(Only one correct answer) 2021

Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by :

- (a) $\frac{2}{3}Ml^2$
- (b) $\frac{Ml^2}{12}$
- (c) $\frac{Ml^2}{6}$
- (d) Ml^2

Question 15

(Only one correct answer) 2021

A body rolls down on an inclined plane without slipping. The kinetic energy of rotation is 50 % of its translational kinetic energy. The body is :

- (a) Hollow cylinder
- (b) Solid sphere
- (c) Sold cylinder
- (d) Ring

Question 16

(Integer type question) 2021

Consider a $20\ kg$ uniform circular disk of radius $0.2\ m$. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force $F = 20\ N$ through a massless string wrapped

around its periphery as shown in the figure.

Suppose the disk makes n number of revolutions to attain an angular speed of $50\ rad\ s^{-1}$, The value of n, to the nearest integer, is ..... [Given : In one complete revolution, the disk rotates by $6.28\ rad$]

Question 17

(Integer type question) 2021

A particle of mass $'m'$ is moving in time $'t'$ on a trajectory given by $\overrightarrow{r} = 1O\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$. Where α and β are dimensional constants. The angular momentum of the particle becomes the same as it was for $t = 0$ at time $t =$ ..... seconds.

Question 18

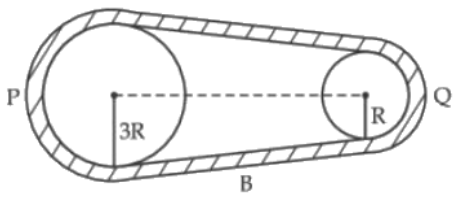
(Integer type question) 2021

A force $\overrightarrow{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of $x = 2$ plane and x-axis. The magnitude of torque of this force about a point $(2, 3, 4)$ is ..... (Round off to the Nearest Integer)

Question 19

(Integer type question) 2021

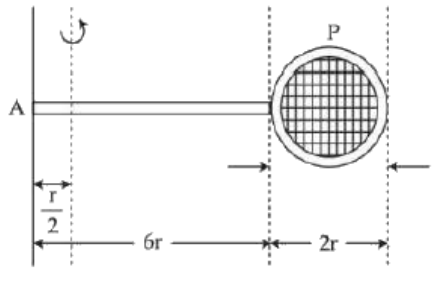
In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_P}{I_Q}\right)$ will be $x : 1$. The value of x will be .....



Question 20

(Integer type question) 2021

Consider a badminton racket with length scales as shown in the figure.

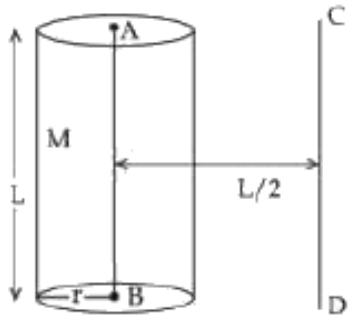


If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be ..... Mr^2.

Question 21

(Only one correct answer) 2021

The solid cylinder of length $80\ cm$ and mass M has a radius of $20\ cm$. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel of AB as shown in figure is $2.7\ kg\ m^2$.

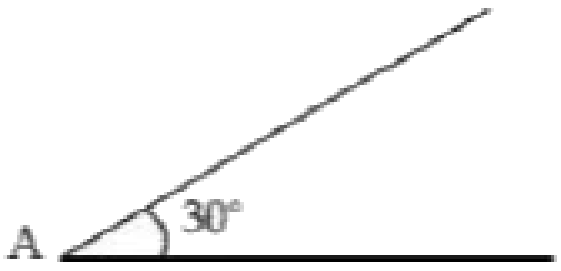


- (a) $7.5 \times 10^2\ kg/m^3$
- (b) $7.5 \times 10^1\ kg/m^3$
- (c) $14.9\ kg/m^3$
- (d) $1.49 \times 10^2\ kg/m^3$

Question 22

(Only one correct answer) 2021

A sphere of mass $2\ kg$ and radius $0.5\ m$ is rolling with an initial speed of $1\ ms^{-1}$ goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A ?



- (a) $0.60\ s$
- (b) $0.57\ s$
- (c) $0.52\ s$
- (d) $0.80\ s$

Question 23

(Only one correct answer) 2021

Consider a uniform wire of mass M and L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing though the center is :

- (a) $\frac{1}{4}\frac{ML^2}{\pi^2}$
- (b) $\frac{ML^2}{\pi^2}$
- (c) $\frac{2}{5}\frac{ML^2}{\pi^2}$
- (d) $\frac{1}{2}\frac{ML^2}{\pi^2}$

Question 24

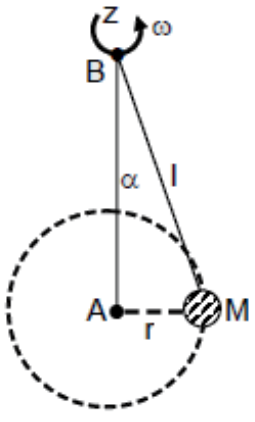
(Integer type question) 2021

A body rotating with an angular speed of $600\ rpm$ is uniformly accelerated to $1800\ rpm$ in $10\ sec$. The number of rotations made in the process is .....

Question 25

(Only one correct answer) 2021

A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about point B is L_B. The correct statement for this system is



- (a) L_A is constant, both in magnitude and direction
- (b) L_B is constant in direction with varying magnitude
- (c) L_A and L_B are both constant magnitude and direction
- (d) L_B is constant, both in magnitude and direction

Question 26

(Only one correct answer) 2021

List –I	List –II

(a)	MI of the rod (length L, Mass M, about an axis $\perp$ to the rod passing through the mid point)	(i)	$8ML^2/3$
(b)	MI of the rod (length L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	(ii)	$ML^2/3$
(c)	MI of the rod (length 2L, Mass M, about an axis $\perp$ to the rod passing through its midpoint)	(iii)	$ML^2/12$
(d)	MI of the rod (length 2L, mass 2M, about an axis $\perp$ to the rod passing through one of its end	(iv)	$2ML^2/3$

- (a) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)
- (b) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)
- (c) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)
- (d) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

Question 27

(Integer type question) 2021

The following bodies

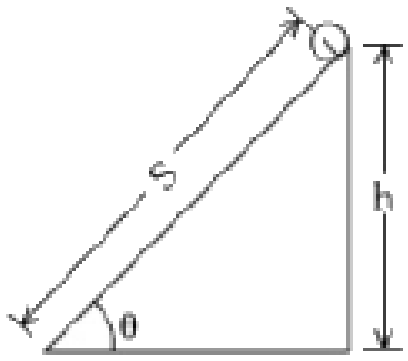
(1) a ring

(2) a disc

(3) a solid cylinder

(4) a solid sphere

Of same mass $'m'$ and radius $'R'$ are allowed to roll without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ..... [Mark the body as per their respectively numbering given in the questions]



Question 28

(Only one correct answer) 2021

Four identical solid spheres each of mass m and radius a are placed with their centers on the four corners of a square of side b. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

- (a) $\frac{8}{5}ma^2 + mb^2$

(b) $\frac{4}{5}ma^2$

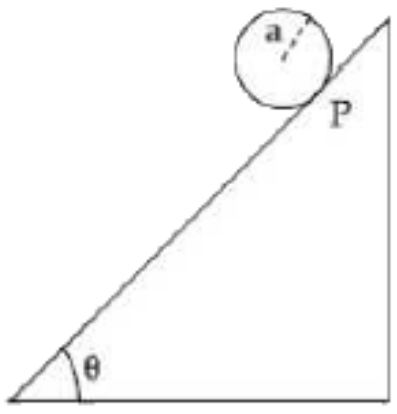
(c) $\frac{4}{5}ma^2 + 2mb^2$

(d) $\frac{8}{5}ma^2 + 2mb^2$

Question 29

(Integer type question) 2021

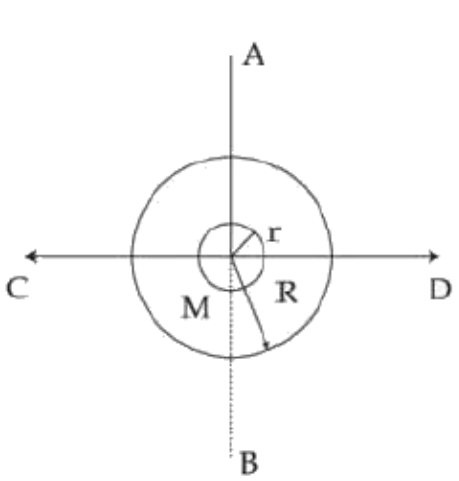
A solid disc of radius $'a'$ and mass $'m'$ rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$. where b is ..... (Round off to the Nearest Integer) (g = acceleration due to gravity, θ = angle as shown in figure)



Question 30

(Only one correct answer) 2021

The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (which is $\perp$ to the plane) of the disc and passing through its center) to MI of smaller disc around one of its diameters lying on its plane ? Given $'M'$ is the mass of the larger disc. (MI stands for moment of inertia)



(a) $2R^4 : r^4$

(b) $2R^2 : r^2$

(c) $R^2 : r^2$

(d) $2r^4 : R^4$

Question 31

(Integer type question) 2021

The angular speed of truck wheel is increase from $900\ rpm$ to $2460\ rpm$ in 26 seconds. The number of revolutions by the truck engine during this time is .....

Question 32

(Integer type question) 2021

A uniform thin bar of mass $6\ kg$ and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the center of mass and perpendicular to the plane of hexagon is ..... $\times 10^{-1}\ kg\ m^2$.

Question 33

(Only one correct answer) 2021

Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Moment of inertia of a circular disc of mass $'M'$ and radius $'R'$ about $X,\ Y$ axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be $I_x,\ I_Y$ & I_Z respectively. The respective radii of gyration about all the three axes will be the same.

Reason R : A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below :

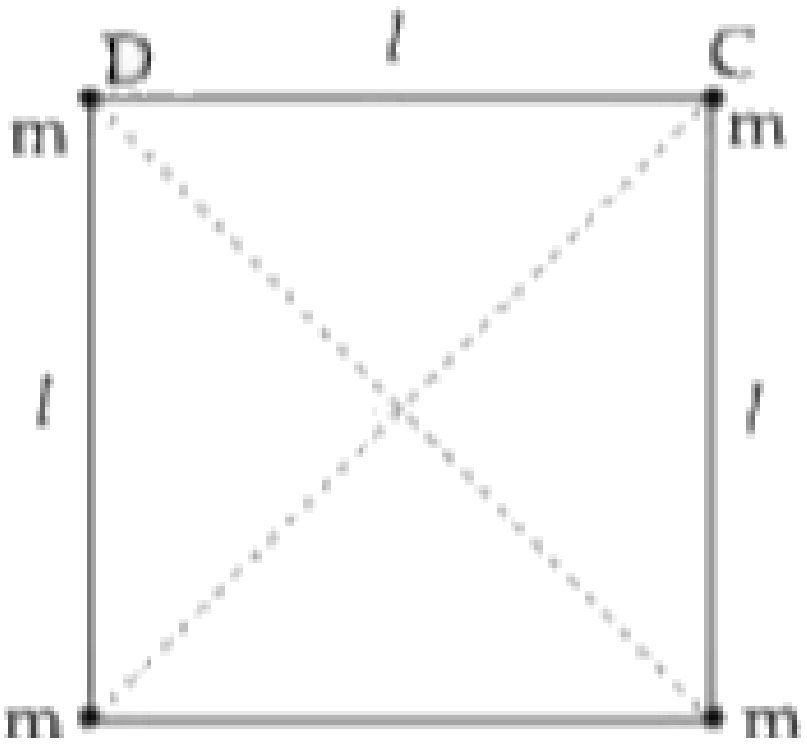
- (a) Both A and R are correct and R is the correct explanation of A.
- (b) Both A and R are correct but R is NOT the correct explanation of A.
- (c) A is not correct but R is correct.
- (d) A is correct but R is not correct.

Question 34

(Only one correct answer) 2021

Four equal masses, m each are placed at the comers of a square of length (l) as shown in the figure.

The moment of inertia of the system about an axis passing through A and parallel to DB would be :



- (a) $\sqrt{3}ml^2$
- (b) $3ml^2$
- (c) ml^2
- (d) $2ml^2$

Question 35

(Only one correct answer) 2021

Moment of inertia $(M.I.)$ of four bodies, having same mass and radius, are reported as :

$I_1 = M.I.$ of thin circular ring about its diameter,

$I_2 = M.I.$ of circular disc about an axis perpendicular to disc and going through the center,

$I_3 = M.I.$ of solid cylinder about its axis and

$I_4 = M.I.$ of solid sphere about its diameter

- (a) $I_1 + I_3 < I_2 + I_4$
- (b) $I_1 = I_2 = I_3 < I_4$
- (c) $I_1 = I_2 = I_3 > I_4$
- (d) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$

Answer 1

Correct answers is C

Solution:

Angular momentum conservation

$I_1\omega_1 - I_2\omega_2 = (I_1 + I_2)\omega$

$\omega = \dfrac{I_1\omega_1 - I_2\omega_2}{I_1 + I_2}$

$\text{Loss} = \dfrac{1}{2}I_1\omega_1^2 + \dfrac{1}{2}I_2\omega_2^2 - \dfrac{1}{2}(I_1 + I_2)\omega^2$

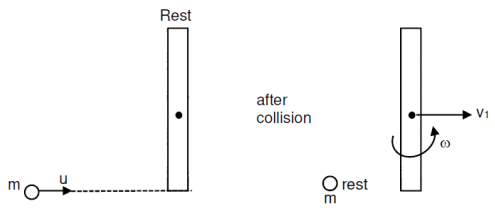
$= \dfrac{1}{2}I_1\omega_1^2 + \dfrac{1}{2}I_2\omega_2^2 - \dfrac{1}{2}(I_1 + I_2)\left(\dfrac{I_1\omega_1 - I_2\omega_2}{I_1 + I_2}\right)^2$

$= \dfrac{1}{2}\dfrac{I_1I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$

$E_i - E_f = \dfrac{I_1I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$

Answer 2

Solution:



Conservation of angular momentum about center of mass of rod

$$mu\left(\frac{L}{2}\right) = \frac{ML^2}{12}(\omega) \text{ ...(i)}$$

$$mu = Mv_1 \text{ ...(ii)}$$

$$1 = \frac{v_1 + \omega\frac{L}{2}}{u} \text{ ...(iii)}$$

Putting v_1 from (ii) and ωL from (i) in (iii)

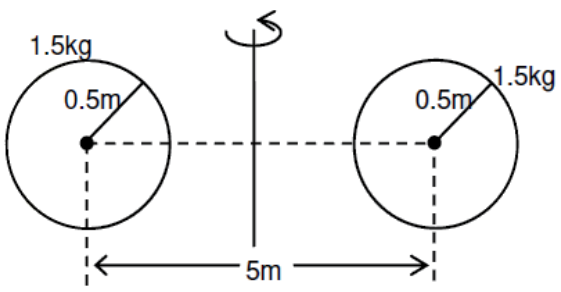
$$u = \frac{m}{M}u + \frac{6mu}{2M}$$

$$1 = \frac{4m}{M}; m/M = 1/4$$

Answer 3

Correct answers is B

Solution:



$$I = \left(\frac{2}{5}MR^2 + Mx^2\right) \times 2$$

$$= \left(\frac{2}{5} \times 1.5 \times (0.5)^2 + 1.5 \times (2.5)^2\right) \times 2$$

$$= [0.15 + 9.375] \times 2 = 1.905\ kg\ m^2$$

Answer 4

Solution:

When disc slides $a_1 = g\sin\theta$

So, $S = ut_1 + \frac{1}{2}a_1t_1^2 = \frac{1}{2}g\sin\theta.t_1^2$ ...(i)

When disc do pure rolling $a_2 = \frac{g\sin\theta}{1 + k^2/R^2} = \frac{g\sin\theta}{1 + 1/2} = \frac{2}{3}g\sin\theta$

So, $S = ut_2 + \frac{1}{2}a_2t_2^2 = \frac{1}{2}.\frac{2}{3}g\sin\theta.t_2^2$ ...(ii)

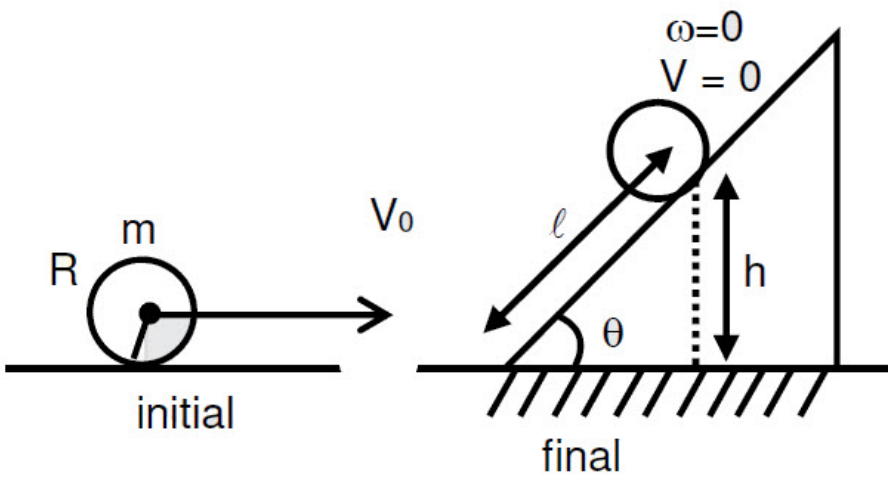
From (i) and (ii)

$$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$$

Answer 5

Correct answers is A

Solution:



$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mV_0^2\left(1+\frac{2}{5}\right) = mgh$$

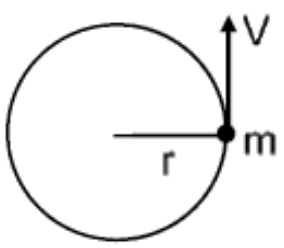
$$h = \frac{7}{10}\frac{V_0^2}{g}$$

$$l = \frac{h}{\sin\theta} = \frac{7}{10}\frac{V_0^2}{g\sin\theta}$$

Answer 6

Correct answers is C

Solution:



$$\overrightarrow{L} = \overrightarrow{r} \times \overrightarrow{p} = \overrightarrow{r} \times m\overrightarrow{v}$$

Answer 7

Solution:

If we consider both bodies have same mass

$K.E._{\text{ring}} = K.E._{\text{Solid cylinder}}$

$$\frac{1}{2}M_R V_R^2 + \frac{1}{2}I_R\omega_R^2 = \frac{1}{2}M_C V_C^2 + \frac{1}{2}I_C\omega_C^2$$

$[I_R = M_R R^2,\ I_C = 1/2M_C R^2]$

$$2M_R V_R^2 = \frac{3}{2}M_C V_C^2 \text{ [considering } M_R = M_C]$$

$$\frac{V_R}{V_C} = \frac{\sqrt{3}}{2}$$

Answer 8

Correct answers is C

Solution:

Apply conservation of energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}m(r^2\omega^2) + \frac{1}{2}I\omega^2$$

$$\omega^2 = \frac{2mgh}{mr^2 + I}$$

Answer 9

Solution:

$$\omega = \frac{600 \times 2\pi}{60} = 20\pi\ rad/s$$

$$\omega_f = \omega_i + \alpha t$$

$$0 = 20\pi - \alpha(10)$$

$$\alpha = 2\pi\ rad/s^2$$

$$\tau = I \times \alpha = \frac{mR^2}{2} \times 2\pi$$

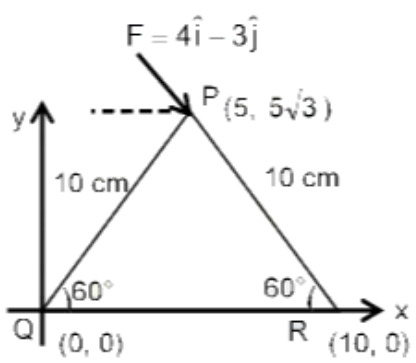
$$\tau = \frac{10 \times 0.04}{2} \times 2\pi = 4 \times 10^{-1}\pi$$

$$x = 4$$

Answer 10

Correct answers is C

Solution:



$$\overrightarrow{\tau} = \overrightarrow{r} \times \overrightarrow{F}$$

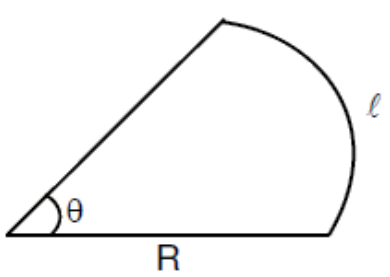
$$\overrightarrow{\tau_Q} = (5\hat{i} + 5\sqrt{3}\hat{j}) \times (4\hat{i} - 3\hat{j}) = (-15 - 20\sqrt{3})\hat{k}$$

$$\overrightarrow{\tau_R} = (-5\hat{i} + 5\sqrt{3}\hat{j}) \times (4\hat{i} - 3\hat{j}) = (15 - 20\sqrt{3})\hat{k}$$

Answer 11

Correct answers is B

Solution:



$l = 4.4\ ly = 4.4 \times 9.46 \times 10^{15}$

length of Arc $= l = R\theta$

$4.4 \times 9.46 \times 10^{15} = R\theta$

$\theta = 4s = 4 \times 4.843 \times 10^{-6}$

$= 1.94 \times 10^{-5}\ rad$

$4.4 \times 9.46 \times 10^{15} = R \times 1.94 \times 10^{-5}$

$R = 2.1455 \times 10^{21}$ meter

Speed $= 8\ AU = 8 \times 1.5 \times 10^{11}\ m/s$

$= 12 \times 10^{11}\ m/s$

4 revolution means distance $= 4 \times 2\pi R$ meter

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{4 \times 2\pi R}{12 \times 10^{11}}\ ;$$

$$\text{time} = \frac{8 \times 3.14 \times 2.1455 \times 10^{21}}{12 \times 10^{11}}$$

time $= 4.5 \times 10^{10}\ sec$

Answer 12

Correct answers is B

Solution:

As there is no external torque

So, angular momentum remain conserved

$$I_1\omega_1 = I_2\omega_2$$

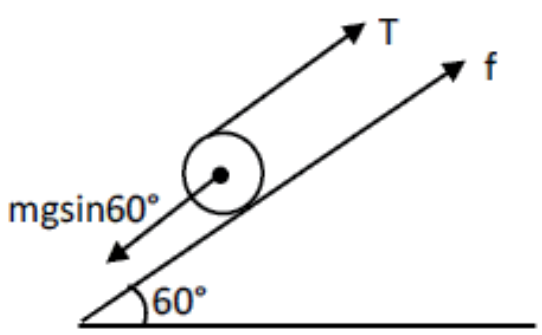
$$(MR^2)\omega = (MR^2 + 2mR^2)\omega'$$

$$\omega' = \frac{M\omega}{M + 2m}$$

Answer 13

Correct answers is A

Solution:



In equilibrium

$$\tau_0 = 0$$

$$T \times R - f \times R = 0$$

$$T = f$$

$$T + f = mg\sin 60^\circ$$

$$f = \frac{mg}{2}\sin 60^\circ$$

$$f = \frac{\sqrt{3}}{4}mg$$

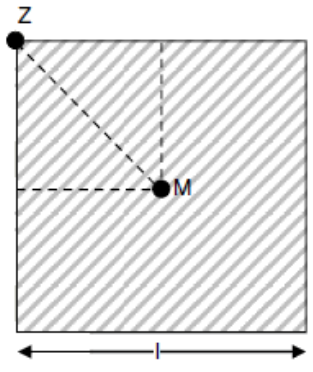
$$\text{but} \quad f_{\max} = \mu mg\cos 60^\circ = 0.2\ mg$$

Here $f > f_{\max}$ so, friction force on the cylinder is $f_{\max} = 0.2\ mg$

Answer 14

Correct answers is A

Solution:



$$I_z = I_{cm} + M\left(\frac{l}{\sqrt{2}}\right)^2$$
$$= \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{4Ml^2}{6} = \frac{2Ml^2}{3}$$

Answer 15

Correct answers is C

Solution:

Given $\frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2}mv^2$

as $v = R\omega$ (pure rolling)

$\frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2;$

$I = \frac{1}{2}mR^2$

Thus, solid cylinder.

Answer 16

Solution:

$$\tau = I\alpha$$
$$\implies \quad \alpha = \frac{\tau}{I} = \frac{F \times R}{MR^2} = \frac{2F}{MR}$$
$$\alpha = \frac{2 \times 20}{20 \times 0.2} = 10\ rad/sec^2$$
$$\implies \quad \omega^2 = \omega_0^2 + 2\alpha\theta$$
$$\implies \quad (50)^2 = 2 \times 10 \times \theta$$
$$\implies \quad \frac{2500}{20} = \theta$$
$$\implies \quad \theta = 125\ rad$$

So, number of revolution $= \frac{\theta}{2\pi} \approx 20$

Answer 17

Solution:

$\overrightarrow{r} = 1O\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

$\overrightarrow{v} = 2O\alpha t\hat{i} + 5\beta\hat{j}$

$\overrightarrow{L} = m(\overrightarrow{r} \times \overrightarrow{v})$

$\overrightarrow{L} = m(1O\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}) \times (2O\alpha t\hat{i} + 5\beta\hat{j})$

At $t = 0,\ \overrightarrow{L} = 0$

At Any time t

$\overrightarrow{L} = m(50\alpha\beta t\hat{k} - 100\alpha\beta(t-5)\hat{k})$

$0 = 50m\alpha\beta[t - 2(t-5)]\hat{k}$

$\implies t - 2t + 10 = 0$

$\implies t = 10\ sec$

Answer 18

Solution:

$\overrightarrow{r} = 3\hat{j} + 4\hat{k}$

$\overrightarrow{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{i} = \overrightarrow{r} \times \overrightarrow{F} = (3\hat{j} + 4\hat{k}) \times (4\hat{i} + 3\hat{j} + 4\hat{k})$

$= 16\hat{j} - 12\hat{k}$

$\therefore \quad |\hat{i}| = 20$

Answer 19

Solution:

As tangential velocities are same

$3R \times \omega_P = R \times \omega_Q$

$\frac{\omega_Q}{\omega_P} = 3$

As $\frac{1}{2}I_P\omega_P^2 = \frac{1}{2}I_Q\omega_Q^2$;

$\frac{I_P}{I_Q} = \left(\frac{\omega_Q}{\omega_P}\right)^2 = 9$

Answer 20

Solution:

$$I = \left[\frac{Mr^2}{2} + M\left(7r - \frac{r}{2}\right)^2\right] + \left[\frac{M(6r)^2}{12} + M\left(3r - \frac{r}{2}\right)^2\right]$$

$$= \left[\frac{Mr^2}{2} + M\frac{(169)r^2}{4}\right] + \left[M3r^2 + \frac{M25r^2}{4}\right]$$

$$= \frac{171Mr^2}{4} + \frac{37Mr^2}{4}$$

$$= \frac{208}{4}Mr^2 = 52Mr^2$$

Answer 21

Correct answers is D

Solution:

$$I = \frac{Mr^2}{2} + M(\frac{L}{2})^2$$

$$= \frac{\rho(\pi r^2)Lr^2}{2} + (\rho\pi r^2 l)\frac{L^2}{4}$$

$$= \rho\pi r^2 L\left(\frac{r^2}{2} + \frac{L^2}{4}\right)$$

$$\implies \rho = \frac{I}{\pi r^2 L\left(\frac{r^2}{2} + \frac{L^2}{4}\right)}$$

On putting the values $\rho = 1.49 \times 10^2\ kg/m^3$.

Answer 22

Correct answers is B

Solution:

For solid sphere

$$k = \sqrt{\frac{2}{5}}r$$

$$a = \frac{g\sin\theta}{1 + \frac{k^2}{r^2}} = \frac{g \times \frac{1}{2}}{1 + \frac{2}{5}} = \frac{25}{7}$$

$$S = ut + \frac{1}{2}at^2$$

$$0 = 1t - \frac{1}{2} \times \frac{25}{7}t^2$$

$$t = \frac{14}{25}$$

Answer 23

Correct answers is B

Solution:

Moment of inertia of ring $= MR^2 = M\left(\frac{L}{\pi}\right)^2 = \frac{ML^2}{\pi^2}$

Answer 24

Solution:

$\omega_0 = 600\ rpm = 10 rps$

$\omega = 1800\ rpm = 30 rps$

$$\alpha = \frac{30 - 10}{10} = 2\ rps^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 10 \times 10 + \frac{1}{2} \times 2 \times 100 = 200$$

Answer 25

Correct answers is A

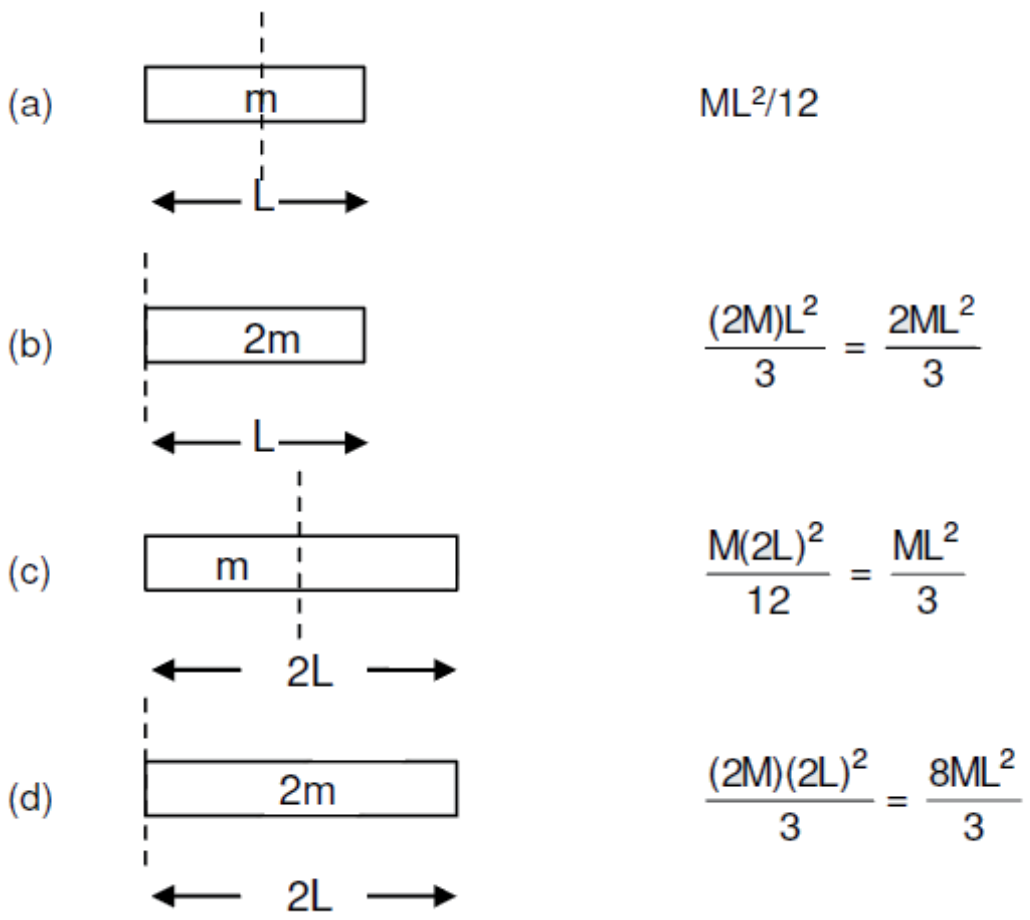
Solution:

Direction and magnitude of angular momentum about A remains same but about B only magnitude of angular momentum remain same but not direction.

Answer 26

Correct answers is D

Solution:



Answer 27

Solution:

Acceleration in pure rolling on an inclined plane is given by

$$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}}$$

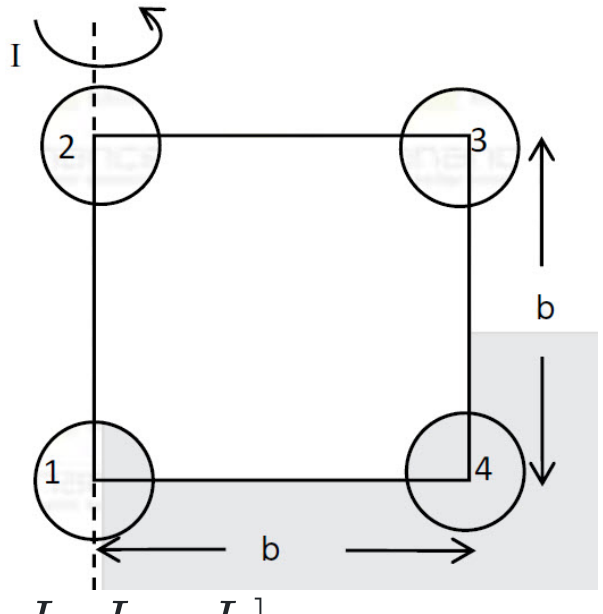
Solid sphere has minimum value of $\frac{K^2}{R^2}$ so maximum value of acceleration will be for solid sphere.

Hence it will take minimum time to reach the bottom

Answer 28

Correct answers is D

Solution:

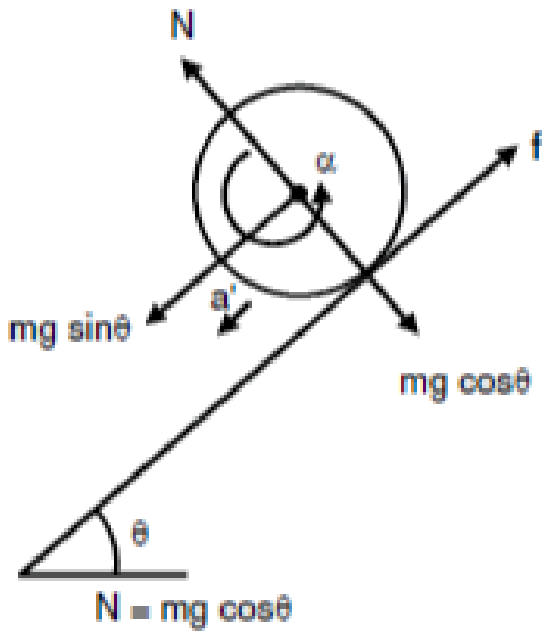


$$I = I_1 + I_2 + I_3 + I_4 \; ; \qquad [I_1 = I_2,\ I_3 = I_4]$$
$$= 2I_1 + 2I_3$$
$$= \left(2 \times \frac{2}{5}ma^2\right) + 2\left(\frac{2}{5}ma^2 + md^2\right) = \frac{8}{5}ma^2 + 2mb^2$$

Answer 29

Solution:

$$mg\sin\theta - f = ma \qquad \ldots(\mathrm{i})$$
$$fa = I\alpha = \frac{ma^2}{2}\left(\frac{a'}{a}\right)$$
$$f = \frac{ma'}{2} \qquad \ldots(\mathrm{ii})$$



From equation (i) & (ii)

$$mg\sin\theta = \frac{3}{2}ma'$$
$$a' = \frac{2}{3}g\sin\theta$$

Answer 30

Correct answers is A

Solution:

Let mass density be σ

$M_1 = \pi R^2 \sigma,\ M_2 = \pi r^2 \sigma$

$$I_1 = \frac{1}{2} M_1 R^2 = \frac{\pi \sigma R^4}{2}$$

$$I_2 = \frac{1}{4} M_2 r^2 = \frac{\pi \sigma r^4}{4};$$

$$\frac{I_1}{I_2} = \frac{2R^4}{r^4}$$

Answer 31

Solution:

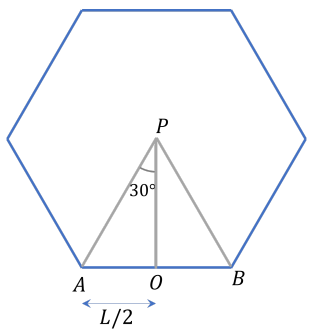
$$\left(\frac{\omega_0 + \omega_f}{2}\right) t = \frac{900 + 2460}{2} \times \frac{26}{60} = 728$$

Answer 32

Solution:

$PO = \dfrac{L}{12 \tan 30^\circ} = \dfrac{\sqrt{3} L}{12}$, where L is the total length of the thin bar $= 2.4$ m

$$M_{AB} = \frac{M}{6} = 1 \text{ Kg}$$



MI of AB about P, $(I_{AB})_P$

$$= \frac{M_{AB}(L/6)^2}{12} + M_{AB}(PO)^2$$

$$= \frac{1(L^2)}{36 \times 12} + 1 \times \left(\frac{3L^2}{12^2}\right)$$

or $(I_{AB})_P = \dfrac{L^2}{36 \times 12} + \dfrac{3L^2}{144}$

Total MI $= 6(I_{AB})_P$

$$= 6\left[\frac{L^2}{36 \times 12} + \frac{3L^2}{144}\right] = \frac{L^2}{72} + \frac{3L^2}{24}$$

$$= \frac{L^2}{72} + \frac{L^2}{8} = \frac{L^2 + 9L^2}{72} = \frac{10L^2}{72}$$

$$= \frac{10 \times 2.4 \times 2.4}{72} = 0.8\ kg\ m^2 = 8 \times 10^{-1}\ kg\ m^2$$

Answer 33

Correct answers is C

Solution:

Correct answer is (c).

Answer 34

Correct answers is B

Solution:

$$I = I_1 + I_2 + I_3 + I_4$$

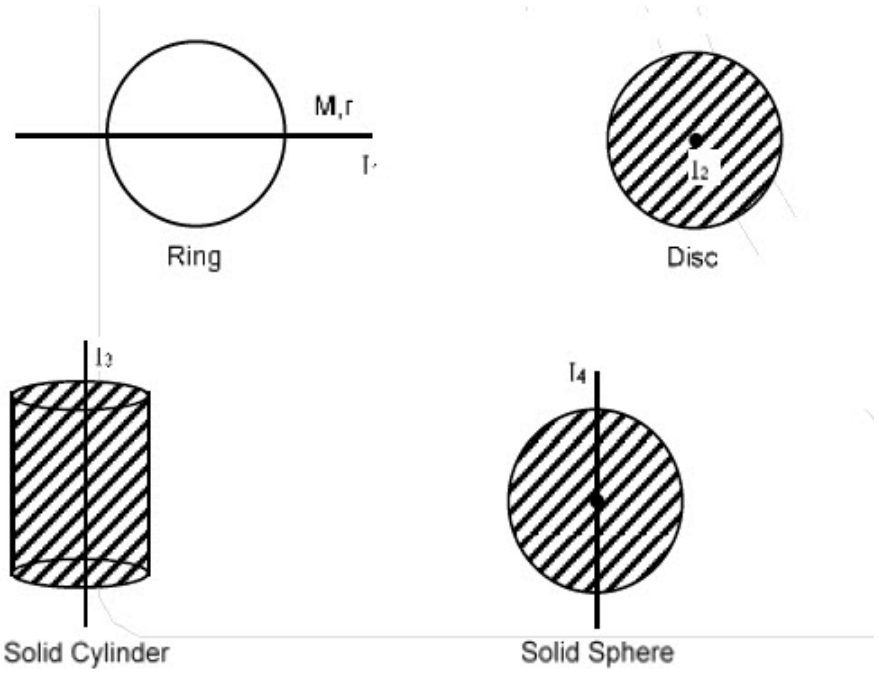
$$= 0 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m(\sqrt{2}l)^2$$

$$I = 3ml^2$$

Answer 35

Correct answers is C

Solution:



$$I_1 = \frac{Mr^2}{2}\ ; \quad I_2 = \frac{Mr^2}{2}\ ; \quad I_3 = \frac{Mr^2}{2}\ ; \quad I_4 = \frac{2}{5}Mr^2$$